# RELIABILITY ANALYSIS OF SPREAD FOOTING OVER CIRCULAR VOID

A thesis submitted in partial fulfillment of

the requirements for the award of

Master of Technology

In

Geotechnical Engineering

By

**K.Venkata Vydehi**

Roll no: 213CE1056

**DEPARTMENT OF CIVIL ENGINEERING**

**NATIONAL INSTITUTE OF TECHNOLOGY, ROURKELA**

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# NATIONAL INSTITUTE OF TECHNOLOGY ROURKELA

## <u>CERTIFICATE</u>

*This is to affirm that the thesis entitled,* **"RELIABILITY ANALYSIS OF SPREAD FOOTING OVER CIRCULAR VOID"** *submitted by* ***K.Venkata Vydehi (Roll No.213CE1056)*** *in partial fulfilment of the requirements for the award of* ***Master of Technology*** *Degree in* ***Civil Engineering*** *with specialization in "Geotechnical Engineering" at* ***National Institute of Technology***, ***Rourkela***, *is an authentic work carried out by her under my supervision and guidance.*

*To the best of my knowledge, the matter embodied in this thesis has not been submitted to any other university/ institute for award of any Degree or Diploma.*

**Dr. Sarat Kumar Das**
Associate professor
Department of Civil Engineering
National Institute of Technology
Rourkela

# ACKOWLEDGEMENT

I take it as privilege to express my gratitude towards my guide, **Dr. Sarat Kumar Das**, Associate Professor Department of Civil Engineering, National Institute of Technology, Rourkela, for his continuous help and encouragement enriched the quality of work.

I express my sincere regards to Prof. **S.K.Sahu**, Professor and Head of Department of Civil Engineering NIT, Rourkela, my faculty advisor, **Prof. S.P.Singh**, Dept. of Civil Engineering NIT, Prof. **S.K. Sarangi**, Director of NIT, Rourkela for giving me the needful resources in the department.

I express my heartful thanks to Mr.Subramaniam and Mr.K.Nagendra for giving their valuable suggestions during the M.Tech program. I would additionally take this opportunity to thank my classmates K.Venkata Sambasivarao, R.Mahendar and Ram M. Desh Pande for their kind co-operation during my project. Many thanks to my friends Rohini, Srikanya, who helped me during my project work.

I expressing my sincere thanks to Mr.Pandit for helped my project work. Finally, I would like to dedicate my work to my family whose constant inspiration and motivation helped me a lot during the successful completion of the project.

(K.V.Vydehi)

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# ABSTRACT

Presence of void beneath the footing affects the stability itself and causes severe damage to the structure. The stability of footing depends on condition of operating  it and the properties of soil, voids. Existence of underground cavities for important and huge structures such as pipe lines and tunnels affects its serviceability. The factors affecting the Bearing capacity and settlement of footing are soil properties, operational conditions, void size, location of void, number of voids and depth of foundation.The performance of footing is analysed over void under different boundary conditions using finite element package PLAXIS 8.1 version. The data used in the present study is taken from available literature. On the other hand, though deterministic analysis gives basic idea, probability of failure of the structure is analysed using reliability method. For this purpose linear response surface model is generated using two-level full factorial design and limit state function is obtained. Reliability analysis is performed on the footing with voids by using first order reliability method (FORM). The probability of failure of structure under standard conditions is analysed and requirement of reliability is discussed.

## ORGANISATION OF THESIS

In Chapter 1 brief introduction to footing over void, causes of formation of void and introduction to usage of reliability analysis also described.

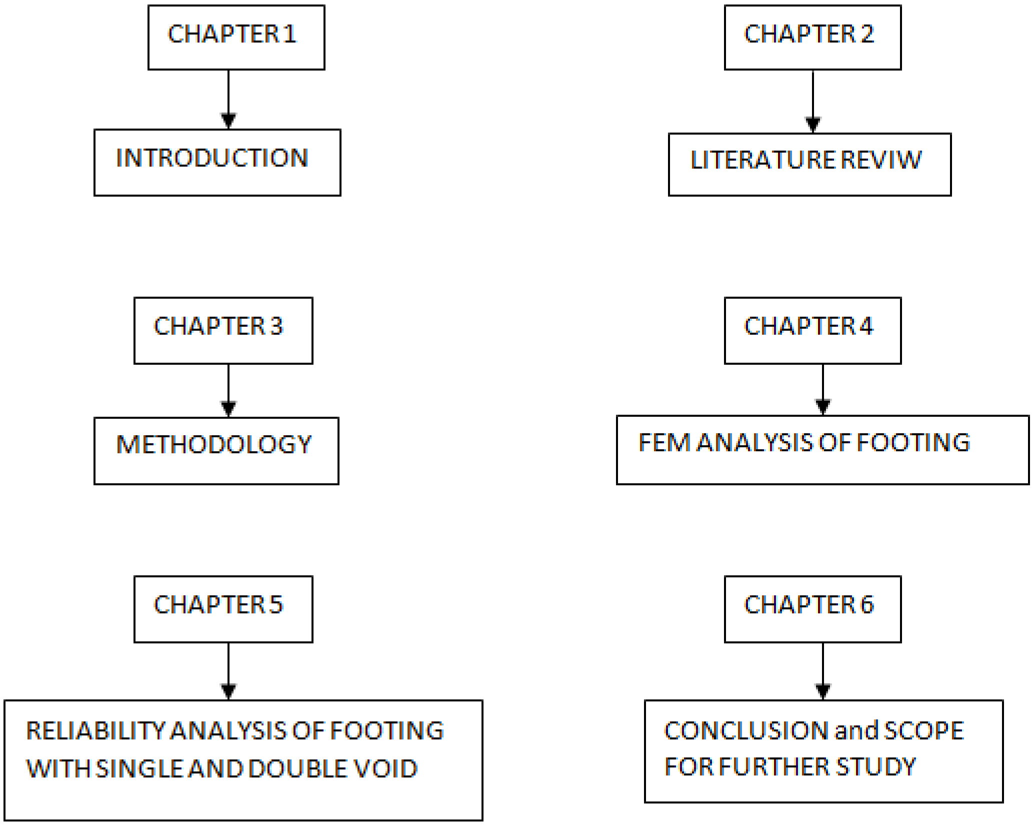
Chapter 2 includes literature review on previous studies carried on analysis of footing over void, experimental and theoretical analysis is described. Review on reliability analysis, response surface method and finite element method are also described.

In Chapter 3 methodology followed for the analysis of present study is described. The theory related to finite element package (PLAXIS 8.1), response surface method is explained. Methods of reliability, Mathematical formulation related to reliability analysis and procedure to solve the problem is described.

Chapter 4 includes analysis of fotting over circular void is analysed using finite element method . Using finite element method the behaviour of footing over void with different boundary conditions, reasons for failure are presented briefly. The finite element results are compared with model testing on footing available from literature.

Chapter 5 includes the reliability analysis of single and double void, Variability in input parameters is considered in reliability analysis. It includes design of experiments, tests and determining the reliability index along with probability of failure of structure under given conditions.

In Chapter 6 conclusions from the various studies and from present analysis are presented and scope for the future study is indicated.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# CHAPTER 1

## 1. INTRODUCTION

### 1.1 INTRODUCTION TO FOOTING OVER VOID

Stability of any structure depends on bearing capacity of foundation soil, which plays major role in Geotechnical engineering. The bearing capacity will change with presence of minerals in soil, with level of water table and with presence of cavities or voids in soil. Existence of underground void affects stability of rigid surface structures such as foundations, rigid pavements over tunnels and underground pipe lines and also the integral stability of structure. Void may exists exactly below the foundation or at any location within the critical region means the region of pressure bulb, it affects stability of footing. From many geological studies the causes and the areas at which voids or cavities formed are as follows:

- In some areas mining activities, blasting causes dynamic loads in soil forms underground voids.
- Construction of tunnels, aqueducts, conduits, underground water tanks and storm or sewer lines in the urban area to reach the utilities of growing population.
- The materials such as salts, dolomite, gypsum and lime stone forms solution by chemical reaction with water or other agents. The space resulted from the flow of this fluid forms cavity at greater frequency inside the ground.
- Cavities may formed based on lithology of rocks and soils.
- Most of voids occur in Calcareous sediments because of their high crushable property and dissolution which is related to flow direction of underground water.
- Methane hydrate is an important constituent in sedimentary rocks of Polar Regions. In methane hydrate large amount of methane trapped in crystal structure of water, forms

ice like solid. Dissociation of this due to temperature changes and other reasons causes formation of voids.

- In storage reservoirs leakage of any substance causes formation of voids.
- Due to differential settlement of buildings, municipal solid waste, poorly compacted backfill and tension cracks in unsaturated cohesive soils, collapse of underground structures such as tunnels, marine subways, tanks and pipes.
- Existence of fault planes in jointed rock mass results cavities.
- From previous studies it is shown that the roads in north have to cross terrains containing ice wedges, due to thermal characteristics of the road surface freezing and thawing of ice takes, which results in formation of holes and dips on or under the surface of road. Due to damage to road it affects the performance of vehicles and motorists. Solution to this problem is time taken and expensive. If thawing and settlement occurs for many years and has stopped at certain time results cavities.
- In stratified soil deposits the different layers can have bearing stratum either softer or stiffer than the underlying stratum results in formation of voids.
- In day to day life septic tanks in residences and water sumps used for storage of drinking water are examples of forms of voids under or at some distance from footing of buildings or other structures. Fig 1.1 shows embankment over cavity, in which the loads from embankment are taken by geomaterials.

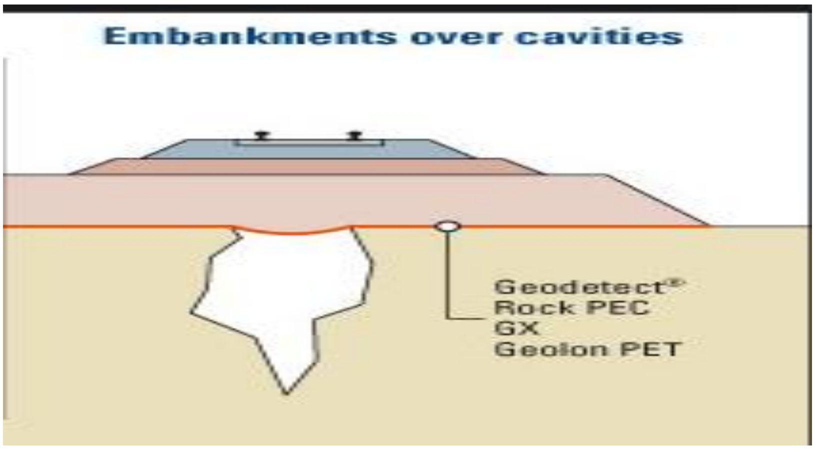



**Fig 1.1 cavity under embankment**

Fig 1.2 shows the extension of voids indifferent directions in stratified soil deposits.

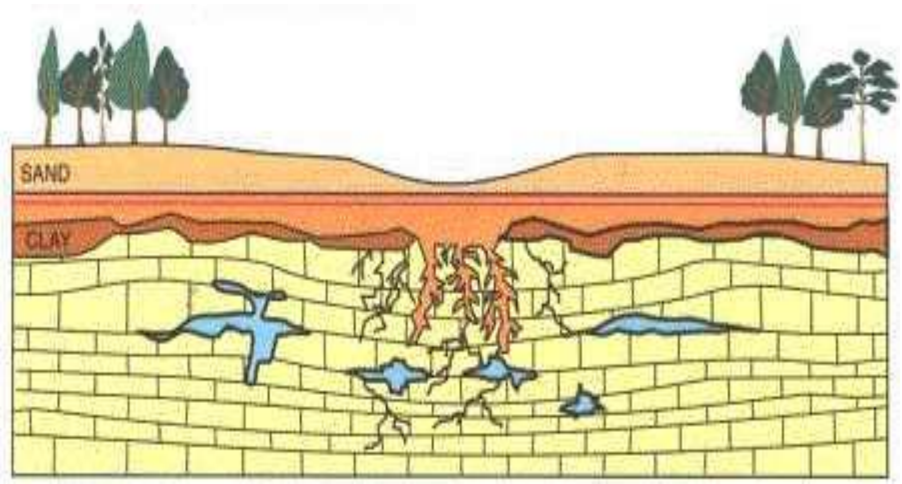



**Fig 1.2  voids in stratified soil deposits**

Practically cavities developed after construction and utilization of structure and are extended continuously on horizontal direction. Therefore voids formed after construction is not accounted for the design of foundation system.  The present study involves in determination of behaviour of footing in terms of bearing capacity and settlement that are affected by presence of void. The analysis includes knowing the critical region under the footing affected by void, considering different factors such as size of void, location of void, depth of foundation and number of voids. As the voids extend in horizontal direction 2 dimensional analysis was performed in present study using finite element package PLAXIS 2D version 8.1

## 1.2. INTRODUCTION TO RELIABILITY

Geotechnical engineering involves study of natural materials. Uncertainities causes variablity in properties of materials. The soil properties determined from experiment in laboratory have slight variation caompared to field data because of experimental errors, calculation errors, instrumental errors, wrong interpretation of data and method of analysis. Even though prabability of failure is high in some cases, in deterministic analysis it shows high factor of safety. Therefore usage of reliability gives the probability of success where as Risk is probability of failure.

## 1.3 OBJECTIVE AND SCOPE OF PRESENT STUDY

The main objective of the present work is to perform the reliability analysis of spread footing over single and double circular void under standard conditions. Scope includes analysis of spread footing over single and double void using FEM under different boundary conditions.

# CHAPTER 2

## 2. LITERATURE REVIEW

The effect of void on the bearing capacity of foundation has been studied over several years and it plays major role in foundation engineering problems. Several methods such as Analytical, Experimental and Numerical studies have been conducted to know the behaviour of footing due to presence of void. Some of studies are discussed as follows. Chowdhury and Grivas (1982) have developed a probabilistic model for progressive failure of slopes. Badie et.al. (1984) investigated stability of spread footing over continuous voids. The model footing tests were performed on kaolinite by considering circular voids for spread and circular footing and results were compared with theoretical analysis using three dimensional finite element program. The bearing capacity and settlement of footing with void for different cases and without void were compared. In this study the depth of footing also taken into account and concluded that stability of footing can be significantly affected when the void is located within the critical region under the footing. Thomas and Billy (1987) developed a mathematical model to design the road embankments with geosynthetics over voids and prersented comparision results by performing various field tests to verify developed mathematical model. Computer analysis was carried out because the mathematical model involves an iterative solution. The study concluded that geosynthetics can be used over voids of 3 m width. Wang et.al. (1987) developed a rational method for stability of footing, complex equations relating the maximum footing pressure and other influencing factors such as void size, location of void and soil strength property. In this study upper bound limit analysis have been used to develop equations for strip footing with continuous void located centrally below footing. From previous results failure mechanisms of foundation soil have been considered for formulating equations for collapse footing pressure. Azam et.al. (1991) investigated the behaviour of strip footing over void supported by a homogeneous soil of finite thickness and a stratified deposit containing two layers. The study was carried out by means of two dimensional finite element analysis by considering circular and rectangular voids with different cases and concluded that the footing performance was affected when depth to bed rock is six times the width of footing in case of homogeneous soil deposits and strength ratio of two layers, top layer thickness in case of stratified soil deposits. Low and Tang (1997) have proposed the procedure to calculate the Hasofer Lind second moment reliability index using spread sheet. Husein et. al (2000) presented the slope stability analysis using first order second moment method and Monte carlo simulation method. The results are compared with conventional methods of slope stability and uncertainities while calculating reliablity index is observed. Gordon et.al (2003) determined the footing width based on settlement obtained through cone penetration test using reliability. Then the actual settlement is compared with finite element method. Schweiger et.al (2005) analysed the performance of simple slope using random variable sets and compared is with finite element method. Variability of material parameters and correlation between the parameters is considered. Kiyosumi et.al. (2007) developed a calculation formula for estimating the yielding pressure of strip footing above multiple voids numerically using two dimensional plane strain finite element analysis. This paper focuses on the nearest void which affects the behaviour of footing than other voids. The authors developed a reduction factor (R) for the yielding pressure of strip footing on the ground with multiple voids based on the reduction factor (R) for a single void by second series of two dimensional finite element analysis. Babu and Basha (2008) have analyzed the sheet pile walls by target reliability approach. Inverse first order reliability method has used to analyze the anchored cantilever sheet pile wall.Sireesh et.al. (2008) conducted model tests to investigate the benefits of provision of geocell reinforced sand mattress over clay with void. The failure of soil was observed similar to punching shear failure. With increase in depth of foundation the thickness of soil layer above the void increased results in improvement in bearing capacity due to soil arching effect. The influencing factors on bearing pressure and settlement of footing are relative density of sand fill, height of geocell layer, base geogrid layer and width of geocell layer. The author concluded that with the increase in above factors the bearing capacity increased substantially. Babu and Srivastava (2010) investigated reliability of four selected earth dams under pseudostatic loading conditions. Variability in soil strenght, seismic coefficient and reservoir full level are considered for the analysis and results are compared with standard reliability methods. Kiyosumi et. al. (2011) were conducted a series of loading test son shallow foundation of sedimentary rock considering square and rectangular voids. Failure mechanisms were found out depending upon whether the void is located exactly below the centre of footing or at an eccentricity from centre to footing. The bearing capacity of footing with void was found out for different cases such as by changing the size of void, location, depth and multiple voids both in horizontal and vertical directions and propagation of slip lines were examined. Subramaniam (2011) analysed slope stability problem, geogrid reinforced footing, anchored sheet pile wall using reliability and finite element method. Reem Sabouni (2013) examined the effect of single and double voids on the settlement and effective stresses underneath the strip footing numerically through parametric study. A study was carried out on size of footing and location of footing below the base. In this paper rectangular void is taken into consideration with the multiple voids located both in horizontal and vertical directions. The settlement and bearing capacities are presented as percentage of no void condition. Lee et.al.(2014) investigated the undrained vertical bearing capacity of strip footing on clay with single and double voids. The undrained bearing capacity factors were determined using design charts by means of finite element analysis.

# CHAPTER 3

## 3. METHODOLOGY

The bearing capacity and settlement of footing over void is analysed using finite element method with available data from literature. The different factors affecting the behavoiur of footing ia analysed and results are compared with literature.

The parameters used for the study are analysed and variability of parameters is analysed which effect the output value. Then design experiments are performed using full factorial design, trail sets are analysed using finite element method. These sets of input and output parameters are analysed using linear surface method in MS Excel. Then limit state function is developed, First order reliability method is used for determining the reliability index. From the relation between probability of failure and reliability index given by USACE chart the probability of failure is determined.

### 3.1 FINITE ELEMENT METHOD

From Field observations it has been noticed that voids extend in horizontal direction continously and that isolated voids are very rare. Therefore for modeling of footing over void 2D idealizations may be feasible. Finite element method provides many options for solving problems concerned with bearing capacity of foundation. The ultimate bearing capacity, settlement and effective stresses generated in the foundation are analysed using Plaxis. Failure mechanism of soil under different loading conditions can be obtained.

**PLAXIS :**

The finite element package available for the geotechnical engineering practitioners is PLAXIS 2D which is user friendly and commercially available. The initiation of this Finite Element Program was held at Delft University of Technology, Netherlands by Pieter vermeer in 1974. Plaxis name was derived from *PL*asticity *AXIS*ymmetry. Because this software is used to perform elastic-plastic calculations for plane strain problems based on high-order elements. later, the code was enriched and could deal with axi-symmetric problems too.

The program uses an implicit time integration scheme, and therefore generates a solution faster than programs that employ explicit time integration schemes. Triangular elements with number of nodes 15 and 6 were used in both 2D and 3D analysis.

**2D finite element model in plaxis:**

Plane strain condition and axisymmetric conditions with two transilational degrees of freedom are available in PLAXIS 2D. In Plane strain condition the strain and displacements in Z-direction are assumed to be zero because the dimension along z-axis is infinity. Depending upon the requirement and type of analysis plane strain condition can be used. In axisymmetric condition the model is symmentrical about central axis.

**Constitutive models :**

**Mohr coulomb model:**

In plaxis the simplest elastic perfectly plastic model is Mohr Coulomb model, which requires five parameters and can be determined from basic soil tests. They are cohesion (C) KN/m2, Young's modulus of elasticity (E) KN/m2, angle of internal friction($\phi$), Piosson's ratio($\nu$) and Dilatancy angle($\Psi$).

**Linear elastic model:**

Linear elastic model follows Hooke's law and requires two parameters namely Young's modulus of elasticity (E), Piosson's ratio($\nu$).

**Mesh properties:**

In plaxis automatic mesh generation will takes place in which the element size varies as very coarse, coarse,medium,fine and very fine. The refinement of line, point and cluster options are available which gives better results.

In Plaxis analysis of footing to know the bearing capacity was performed by staged construction procedure. In the staged construction the load is applied by stages, in increments upto failure of soil takes place. The load taken by soil is represented by $\Sigma$ Mstage which varies from 0 to 1. $\Sigma$ Mstage=1 represents, applied load is taken by soil completely. For example 100KN/$m^2$ load is applied and $\Sigma$ Mstage is 0.6 then the load taken by soil is 0.6*100=60KN/$m^2$ .

## 3.2 RESPONSE SURFACE METHOD

A statistical technique developed for improving and optimizing process through emperical model building by Box and Wilson (1951) called response surface method. The optimization of any value means maximizing or minimisation can be done using RSM by changing the input parameters which are known as random variables. In this method two types of models are used namely (1)Linear surface model and (2) Non linear surface model

To optimize a response with the set of random variables, which is also called as design variables two procedures are followed in response surface method, namely (1) Design of experiments(DoE) and (2) Response surface analysis.

To improve the quality of information the data generating process is manipulated in designed experiment. An experiment consists of series of tests also known as runs in which input parameters are changed to know the changes in output parameters along with cause.

To determine the correlation between variables and objectives locally or globally, available data can be interpolated in response surface analysis. If the data follows a flat surface,it represents the first order model.

The first order model with controlled input parameters $x_1$ and $x_2$ to get a simple response y using response surface method is

$$y=\beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2 + \in$$

Fig 2 shows linear response surface model. In the polynomial equation $\in$ represents error due to uncontrolled parameters and experimental error. The coefficients $\beta_1$ and $\beta_2$ are main effects, while $\beta_{12}$ is a two way interaction effect. In an experiment the coefficients $\beta_0$, $\beta_1$, $\beta_2$ and $\beta_{12}$ calculated accurately by changing $x_1$ and $x_2$ to get a response y.

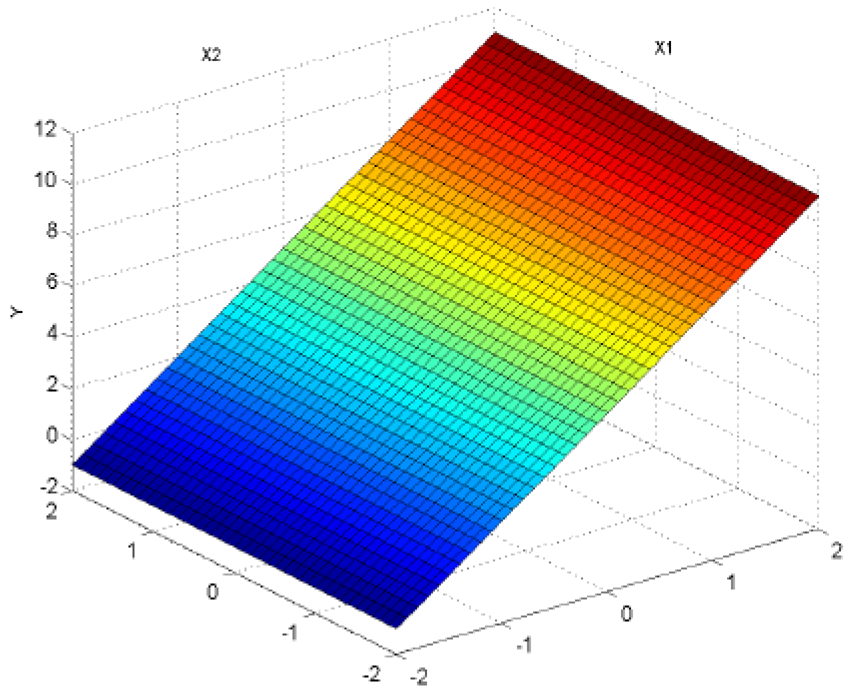



**Fig 3.1 Linear response surface**

A higher order model must be used if there is a curvature data,because the first order model would show a significance of lack of fit. Polynomial model is as follows

$$y= \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2 + \beta_{11} x_1^2 + \beta_{22} x_2^2 + \epsilon$$

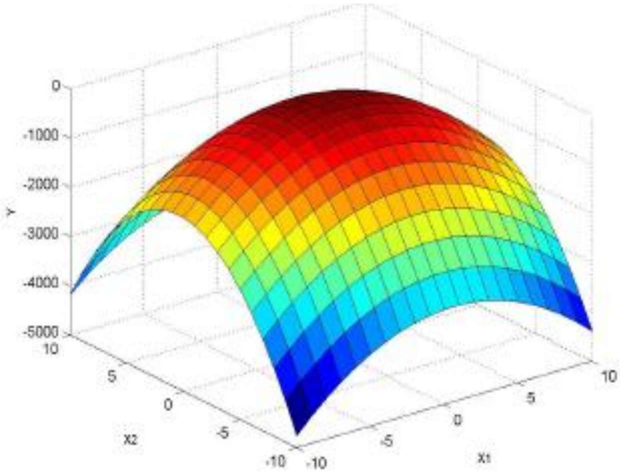



**Fig 3.2 Non-linear response surface**

**DESIGN OF EXPERIMENTS(DoE):**

**Factorial Design:**

In factorial experiment the design variables are varied together, instead of one at a time. Factorial design determines the effect of changing variables on response variable. Each of the n factors has only two levels, therefore the design experiment has $2^n$ experimental trails. These designs are know as $2^n$ experimental trails.

Two- level full factorial design:

$2^2$ factorial design:

The simplest design $2^n$ series with two factors $x_1$and $x_2$ and runs in two levels.

Matlab code for desigh of experiments is

```
>> dFF=ff2n(2)

dFF =

     0     0
```

```
    0     1
    1     0
    1     1
```

here the number of treatments is four which is given by number of rows. Each column represents settings for a single factor with '0' and '1'. If number of parameters in an experiment is 3 then the Matlab code is as follows:

```
>> dFF=ff2n(3)

dFF =

    0     0     0
    0     0     1
    0     1     0
    0     1     1
    1     0     0
    1     0     1
    1     1     0
    1     1     1
```

Here binary values are used for design purpose only and they don't carry any meaning.

For the number of parameters of 3, eight sets of experiments has generated , '0' and '1' are estimated as $\mu+2\sigma$ and $\mu-2\sigma$. $\mu$ is the mean of the variable and $\sigma$ is the standard deviation of corresponding variable.

$\sigma = \mu * cov$

Cov is coefficient of variation of given parameters of the soil. The design sets ($x_1$,$x_2$and $x_3$) are used to conduct the experiments and output response (y) is obtained. Using eight sets of parameters ( input and output) linear or nonlinear regression analysis is performed using

MS Excel.

## 3.3 RELIABILITY ANALYSIS

**Reliability:**

Reliability is the measure of quality of geotechnical structure over a specified time under standard conditions. In other words reliability is probability of success.

**Methods of reliability:**

1. First Order Reliability Method (FORM)

2. Second Order Reliability Method (SORM)

3. Monte Carlo Sampling (MCS)

4. Numerical Integration (NI)

5. Increased Variance Sampling (IVS)

Terminology used in reliability:

Mean(μ):

First central moment which is defined as the average value of data set and measures central tendency of data.

**Variance:**

Second central moment that measures spread in data about mean.

**Coefficient of variation(cov):**

It measures the dispersion of data. Higher value of cov represents the higher diepersion about its mean.

**Covariance:**

Covariance indicates the degree of linear relationship between two random variables (x, y).

$\mathrm{Cov}(x,y) = E((x-m_x)(y-m_y)) = E(xy- m_x m_y) = E(xy) - E(x)E(y)$

The uncertainities in a variable can be quantified using a mathematical model satisfying different functions such as probability density function,probability mass function and cumulative distribution function. Continuous random variable follows normal distribution and beta distribution.

**Properties of Normal distribution:**

1. The parameter varies between –∞ to +∞.
2. It is perfectly symmetric about mean.
3. Mean,Median and Mode values are same.

Normal distribution $f_x(x) = \frac{1}{\sigma x\sqrt{2\pi}} e^{\frac{-1}{2}(\frac{x-\mu x}{\sigma x})^2}$

Reliability is the probability of success and its value is one minus probability of failure (1- $P_f$ ). If ‘R’ is the resistance and ‘S’ is the load on the structure, then the structure will fail if ‘R’ is less than ‘S’ and propability of failure can be expressed as

$P_f$ = P [R≤S] = P [(R-S) ≤0 ]

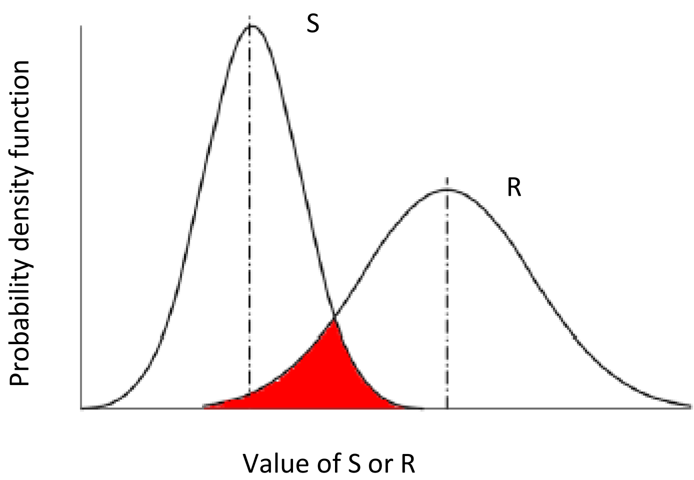



**Fig 3.3 Overlapped area is the probability of failure of random variable R and Q**

the probability of failure is the shaded area of overlapping as shown in the Figure 2.5 and mathematically denoted as

$$Pf = \int_{-\infty}^{+\infty} \mathrm{GR(r)}\ \mathrm{Gs(s)}\ ds$$

Reliability, $\mathrm{R} = \int_{-\infty}^{+\infty} \mathrm{GR(r)Gs(s)}\ ds$

Where GR(r) is CDF of resistance R and Gs(s) is CDF of load S.

Limit state function can be defined as a mathematical model which relates variables such as load and resistance. It is expressed as

$$Z = (R-S) = f(R,S) = f(X_1, X_2, X_3, \ldots, X_n)$$

z = margin of safety

If the limit state function is zero then failure would occur and the equation is known as limit state equation.

i.e., $f(X_1, X_2, X_3, \ldots, X_n) = 0$, defines the safe and unsafe which may be linear or non linear.

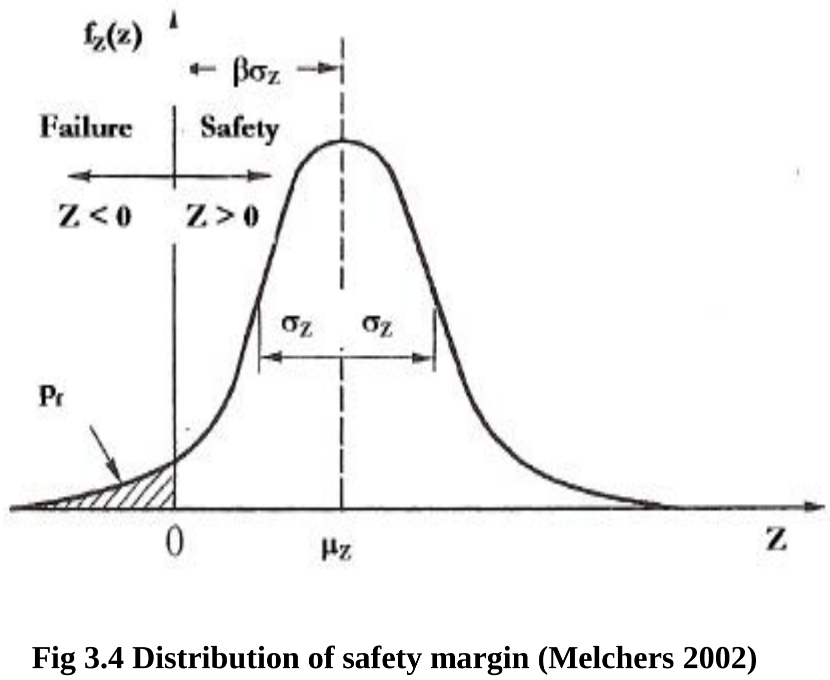



**Fig 3.4 Distribution of safety margin (Melchers 2002)**

Cornell gave expression for reliability index

$\beta = \frac{\mu z}{\sigma z}$ and $P_f = \phi(-\beta)$ ,$\phi$ is CDF of standard normal variable.

### 3.3.1 First order reliability method:

Baecher G.B and Christian J.T. (2003), described The methodology of FORM which is also know as first order second moment method. The load of a system is 'S' and resistance is 'R', these values are uncertain. These values have mean, variance and covariances. The margin of safety which represents performance of function is M = R-S = 0.

The probability of failure is $P_f = P[(R-S) \leq 0]$

Cornel given the reliability index $\beta = \frac{\mu_R - \mu_S}{\sqrt{\mu_R^2 - \mu_S^2}}$

M is a linear function given by $M = b_0 + b_1 x_1 + b_2 x_2 + \ldots\ldots + b_n x_n$

Then $\mu_M = B_0 + \sum_{i=1}^{n} b_i \, \mu_i$

$$\sigma_M^2 = \sum_{i=1}^{n} b_i^2 \, \sigma_i^2 + 2 \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \rho_{ij} \, b_i b_j \sigma_i \sigma_j$$

### 3.3.2 Advanced FORM method:

Hasofer and Lind (1974) has given Advanced FORM method based on geometric analysis. In this method non normal variables must be converted to normal variables,because this method is applicable for normal variables only to get reliability index. The method uses transformed coordinates to represent reliability index, with zero mean and unit standard deviation. Reliability index is defined as the minimum distance between the peak of multivariate distribution of input variables and a limit state function defining the failure surface.

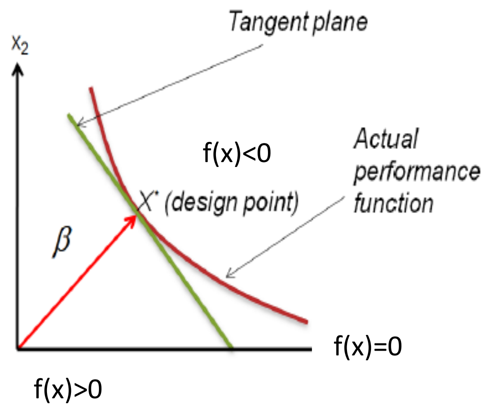



**Fig 3.5 Hasofer Lind reliability index**

$X^*{}_i = \frac{X_i - \mu_i}{\sigma_i}$ ; i= 1,2,3,……,n

The reduced limit state equation is given by f($X_1, X_2, X_3, \ldots, X_n$) = 0

$X^*$ on f(x) is referred as design point. The point of intersection of performance function and tangent plane is most probable point of failure. From fig 3 distance between the origin and $X^*$ is reliability index.

$$\beta = F(X*) = 0^{min}\sqrt{(X*)^t(X*)}$$

$$P_f = \phi(-\beta)$$

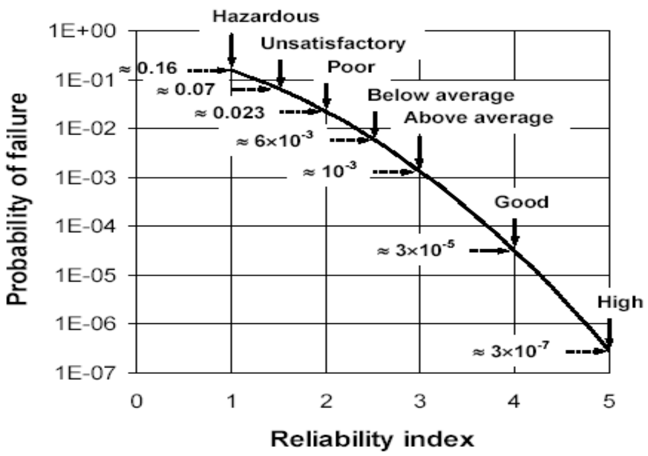



**Fig 3.6 Relationship between Reliability index(β) and Probability of failure(Pf) (USACE 1997)**

# CHAPTER 4

## 4. ANALYSIS OF FOOTING OVER VOID

### 4.1 BASIC CONCEPT

The failure mechanism for the foundation without void is shown in fig 4.1 given by Terzaghi. Limit equilibrium approach is used to know the bearing capacity equation. According to Terzaghi three different failure mechanisms are observed based on pattern of shearing zones as General shear failure, Local shear failure and Punching shear failure.

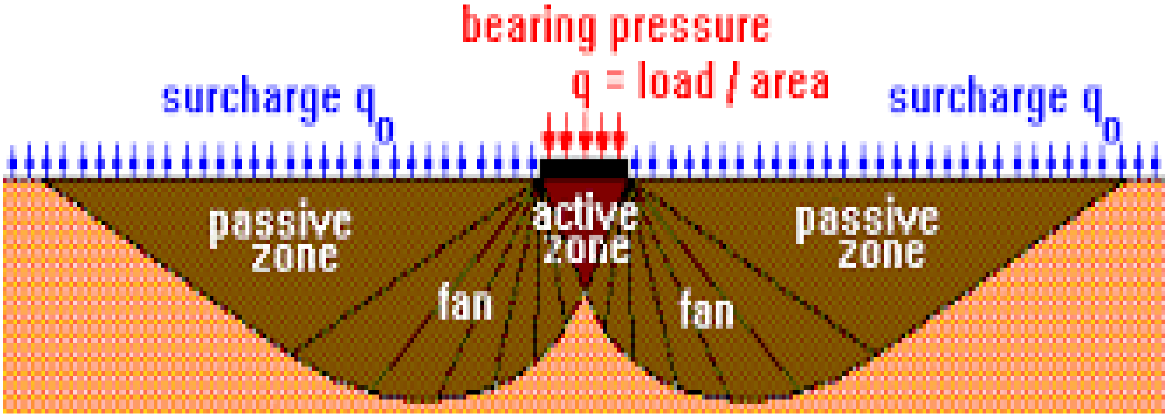



**Fig 4.1 Failure mechanism given by Terzaghi**

Presence of voids under foundation affects bearing capacity and settlement of footing. Many studies have been carried out to know the significant effect on footing. Baus's experimental results gave different failure mechanisms for footing over circular void. These sliding planes depends on several factors such as footing width,size of void, strength parametres and depth to void. Based on Baus's experimental results Wang et.al adopted three failure mechanisms to formulate equations for coolapse load of footing over circular void. Fig 4.2 shows failure machanisms considered by Wang et.al. Baus and Wang (1983) have observed the failure under the footing with void is punching shear failure in which shear planes are confined to soil mass causing collapse of soil into void just below the footing. In figure 4.2 first mechanism is represented by 'mechanism A' and and remaining two mechanisms are represented by 'mechanism B' and 'mechanism C' respectively.

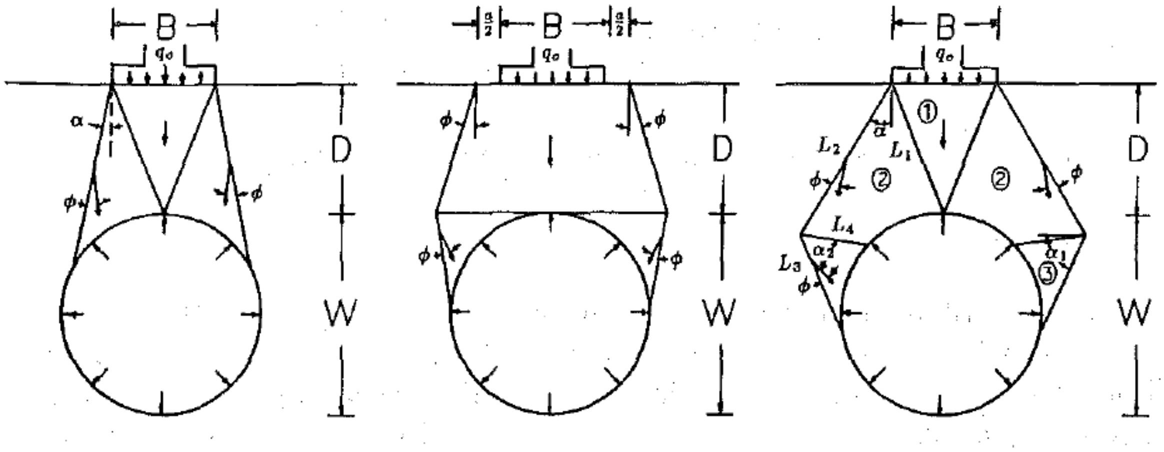



**Fig 4.2 Failure mechanisms for footing collapse (Wang et.al 1986)**

The analysis of footing can be made based on two criteria namely Beraing capcity criteria and settlement criteria. An attempt has been made to know the behavoiur of footing over void using finite elment analysis. The influenced factors are void size, void location and multiple voids. Soil properties and field conditions also a major part of the analysis. In present study the problem was taken from Literature, solved using PLAXIS and compared with literature.

## 4.2 ANALYSIS OF FOOTING OVER VOID IN PLAXIS

The problem was taken from Badie et.al (1984) in which stability of a spread footing was studied over void by model testing. In model testing the soil with void used is commercially available kaolinite with 95% compaction based on standard proctor test. The bearing capacity of footing is compared for two conditions one for the 'no void' and other for the void with

D/B=2 under the footing.The same is modeled and simulated in PLAXIS and results are compared. In present study circular void was approximated by hexagon and the foundation soil mass was represented finite number of descrete elements interconnected by nodal points, Triangular element with 15- nodes was used as soil element. In later section the footing was analysed for different conditions. Fig 4.3 shows the geometrical model of footing as given by Badie et.al (1984).

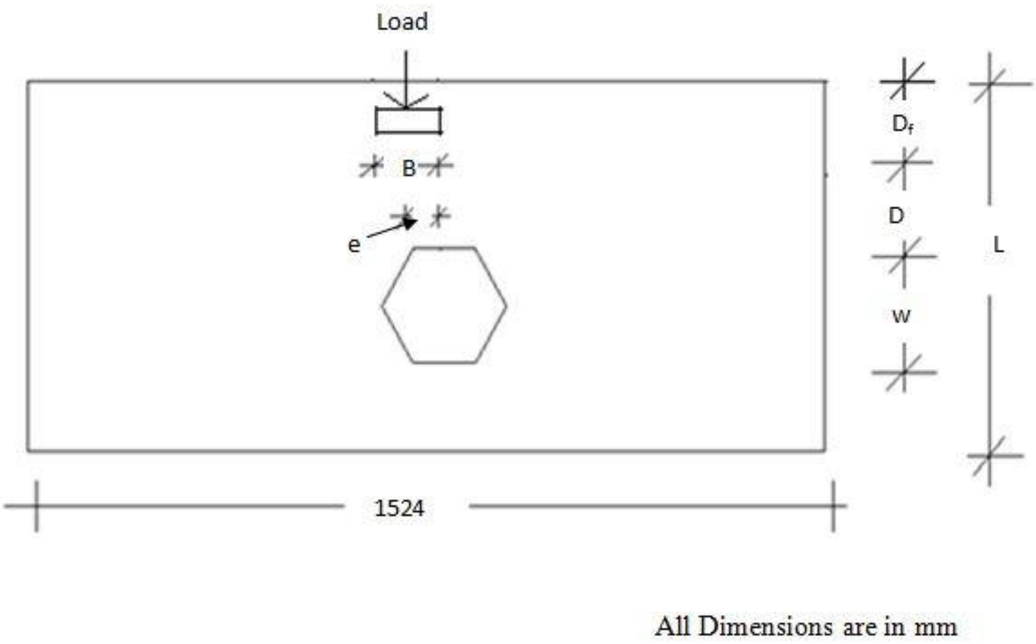


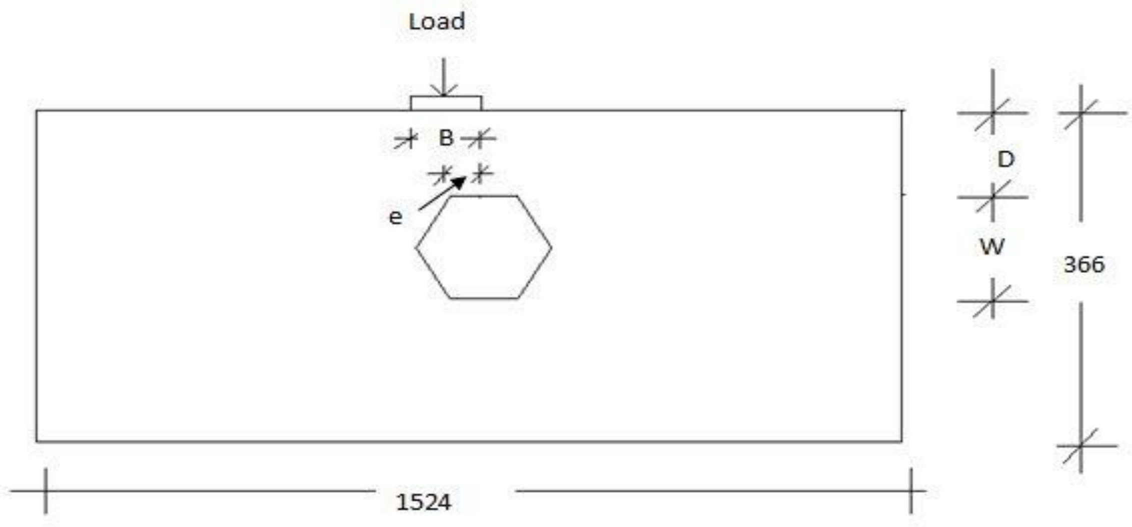

**Fig 4.3 Geometrical model of footing**

Here 'B' is width of footing taken as 2 inches (51mm), thickness of footing is 13 mm, 'W' is size of void, 'D' is the distance between base of footing and top of void, '$D_f$' is depth of foundation and 'e' is eccentricity ( distance between centre of footing and centre of void).

### 4.2.1 BEARING CAPACITY CRITERIA ( $D_f$ =0)

#### 4.2.1.1 Analysis without void

Using finite element method the bearing capacity of footing over circular void is analysed in Plaxis without void and with void by considering voids of different sizes, different locations, depth to void from base of footing, multiple voids and results are compared with literature (Badie). Fig 4.4 shows the geometrical model of foundation with depth of foundation zero. The footing is placed at the surface of soil, therefore the failure planes will reach the suface of soil mass. In this case the footing with no void condition failure surfaces reaches the ground surface. The material properties used for soil and footing are presented in table 1.




All Dimensions are in mm.

**Fig 4.4 Geometrical model of footing with Df =0**

**Table 1 Soil Parameters**

| **Description** | | Soil | steel |
|---|---|---|---|
| Unit weight ( $kN/m^3$ ) | $\gamma$ unsat | 13.23 | 7596 |
| | $\gamma$ sat | 16.28 | |
| Elasticity modulus(kPa) | | 19872 | $200 \times 10^6$ |
| Effective poisson's ratio | | 0.39 | 0.28 |
| Effective cohesion (kPa) | | 158.7 | $1.24 \times 10^5$ |
| Effective friction angle ( $^0$) | | 8 | 0 |

In plaxis after creating geometry model, material properties are assigned and mesh is generated. Initial effective stresses are generated then load is applied in calculation phase until failure of footing takes place. Fig 4.5 shows the soil model in Plaxis without void. Fig 4.6 shows the mesh generated in Plaxis. Fig 4.7 shows the failure mechanism of footing under no void condition interms of incremental strains with depth of foundation zero, which resembles failure pattern given by Terzaghi.

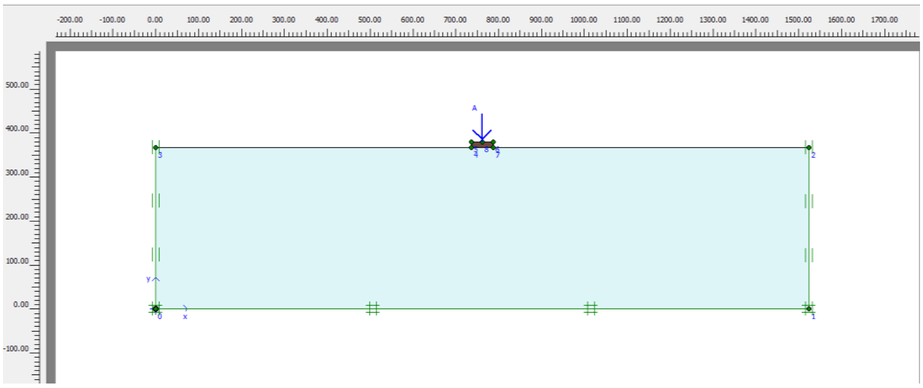



**Fig 4.5 soil model in PLAXIS**

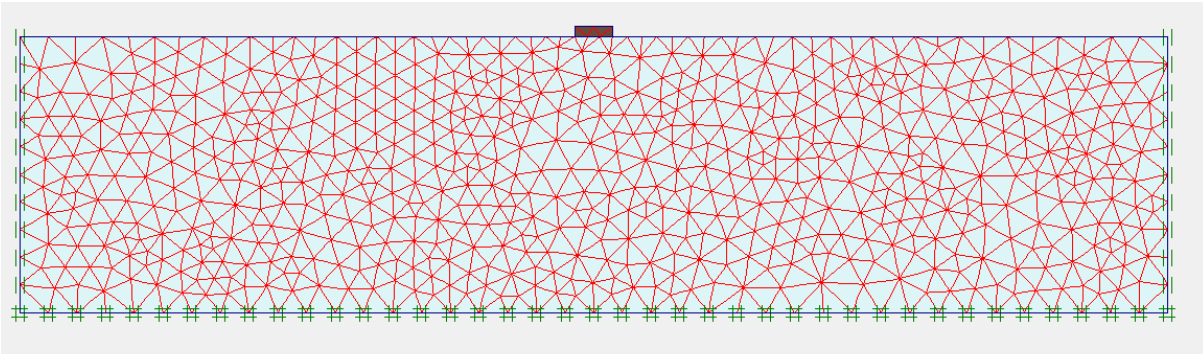


**Fig 4.6 Generated mesh in Plaxis**

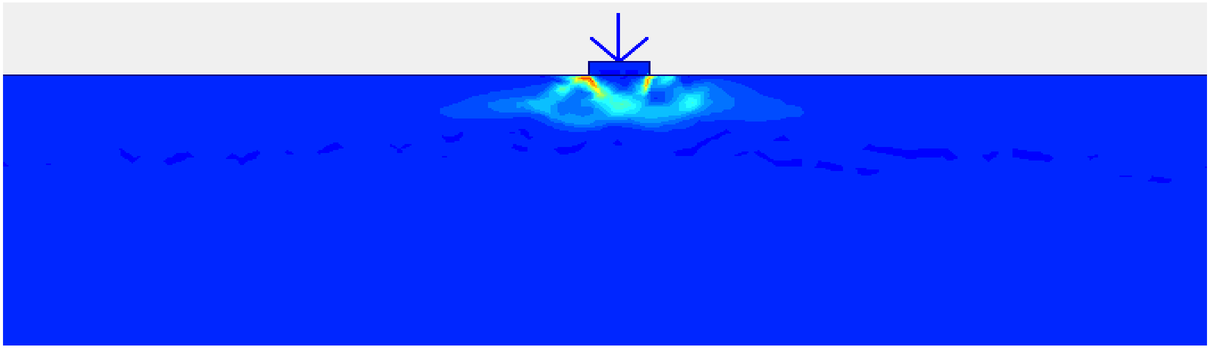


**Fig 4.7 Incremental strains with no void**

Fig 4.8 shows the Σ Mstage Vs Displacement curve from Plaxis. The load at which soil fails is taken as ultimate load of footing. Here the value of Σ Mstage has not reached '1', its value is 0.0634 as the soil fails without taking the complete load. The bearing capacity was obtained from load displacement curve following the procedure given in IS 1888. The graph is drawn between load and displacement on X and Y axis respectively with log-log scale. As per IS1888 the point at which the slope of the curve becomes minimum constant value is taken as bearing capacity of foundation. The bearing capacity obtained from Plaxis is 1243 kPa.

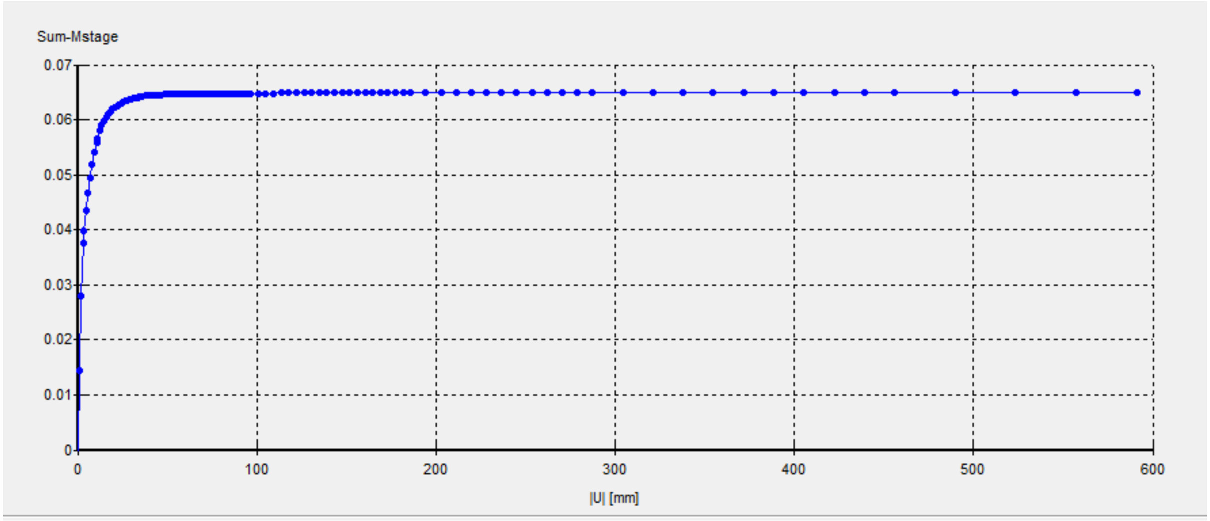



**Fig 4.8 Σ Mstage Vs Displacement curve for no void condition**

### 4.2.1.2 Analysis with single void variation in e/b and D/B

Badie et.al (1984) conducted several experiments to know the behaviour of footing considering circular voids at different locations. Fig 4.9 shows the footing over void with e/B=0; D/B=2; W/B=2.4; $D_f$/B=0 modelled in Plaxis. For the analysis the portion representing void is deactivated, initial stresses are generated and calculation phases are done.

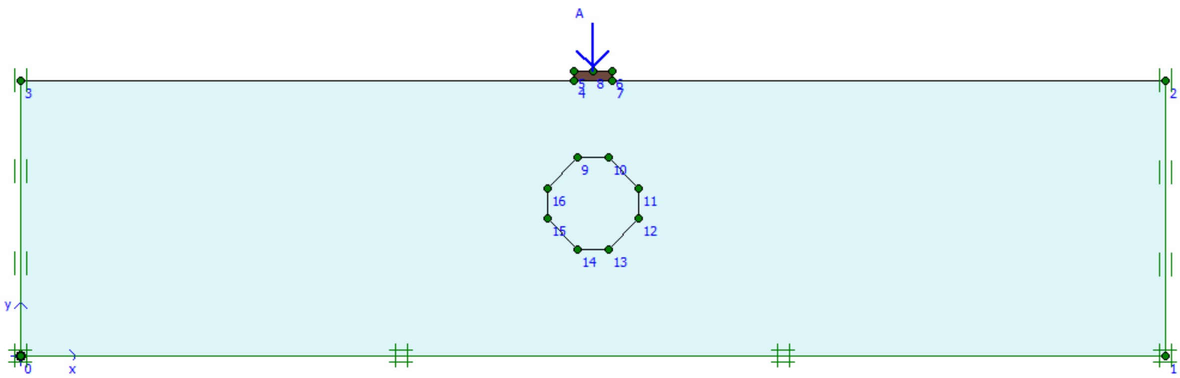


**Fig 4.9 Footing over void with D/B=2; e/B=0(Plaxis model)**

Fig 4.10 shows the total displacements of soil at failure and which resembles punching shear failure of soil. The soil immediately below the footing undergoes maximum displacement because the soil mass underneath collapsing into the void.

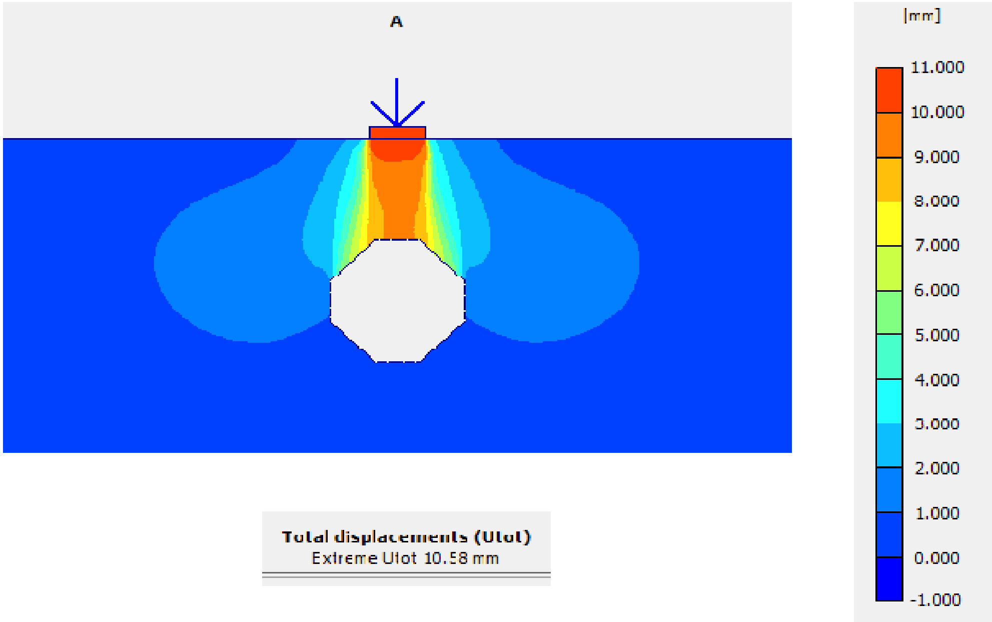



**Fig 4.10 Total displacements of soil (e/B=0; D/B=2)**

Fig 4.11 shows the failure mechanism of footing with void. The failure mechanism obtained from Plaxis is matched with mechanism B given by Wang et.al (1987).

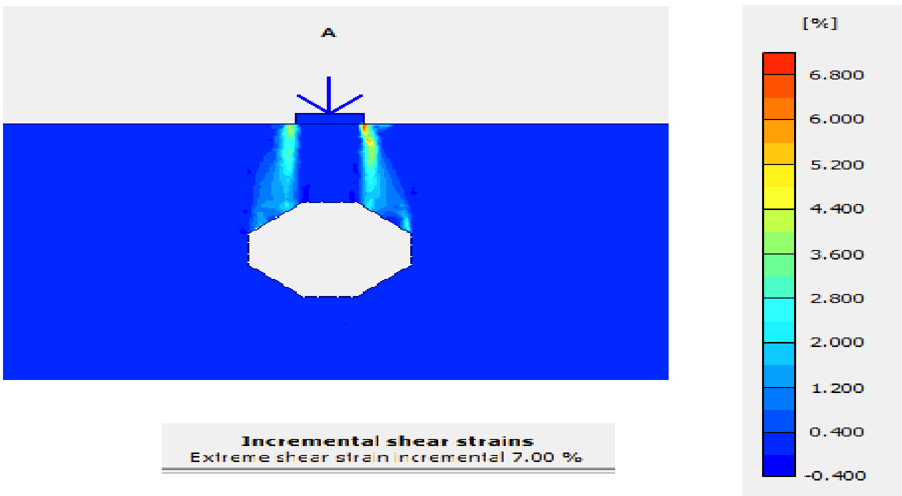



**Fig 4.11 Incremental strains (e/B=0; D/B=2)**

Fig 4.12 shows the Σ Mstage Vs Displacement curve from finite element method. The load is applied at incremental rates until the failure. The load at which soil fails is taken as ultimate load of footing.

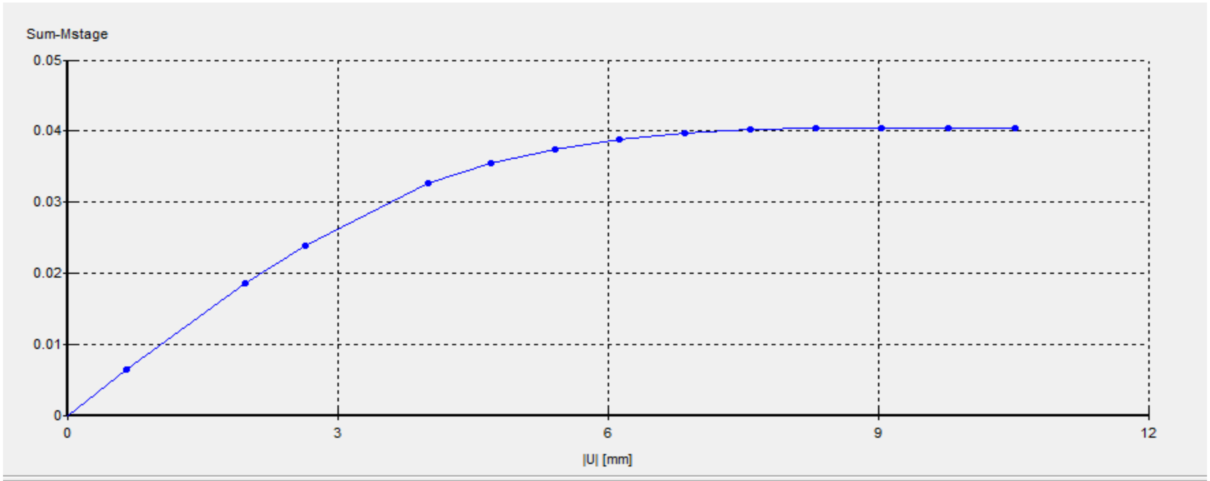



**Fig 4.12 Σ Mstage Vs Displacement curve for void condition**

The bearing capacity calculated from Plaxis for footing with void condition is 788 kPa. From literature the bearing capacities are compared and are shown in table2.

**Table 2 Comparison of Bearing capacity of footing for different conditions between literature and present study**

| Method | Bearing capacity (kPa) | |
|---|---|---|
| | No void condition | With Void (e/B=0; D/B=2) |
| Testing on Model footing (Badie et.al 1984) | 1250 | 817 |
| Finite element analysis-Plaxis (present study) | 1243 | 788 |

For a given values of e/B ( e/B = 0,1,2,3,4)and D/B (D/B= 1,2,3,4) the bearing capacities are obtained with W/B=2.4 and $D_f$/B=0. In present analysis B is width of footing (51mm), $D_f$ is depth of foundation and W is size of void (122.4mm). Fig 4.13 shows variation of bearing capacity of footing with and without void conditions. The bearing capacity is increasing with the increase in e/B and D/B ratio. With further increase in e/B and D/B ratios the bearing capacity reaches the value of no void condition. For a given value of e/B the rate of increase in bearing capacity increases first and then decreases with increase in depth of void from base of footing. The curve obtained is in bowl shape. Here the problem is analysed with zero depth of foundation and w/B=2.4.

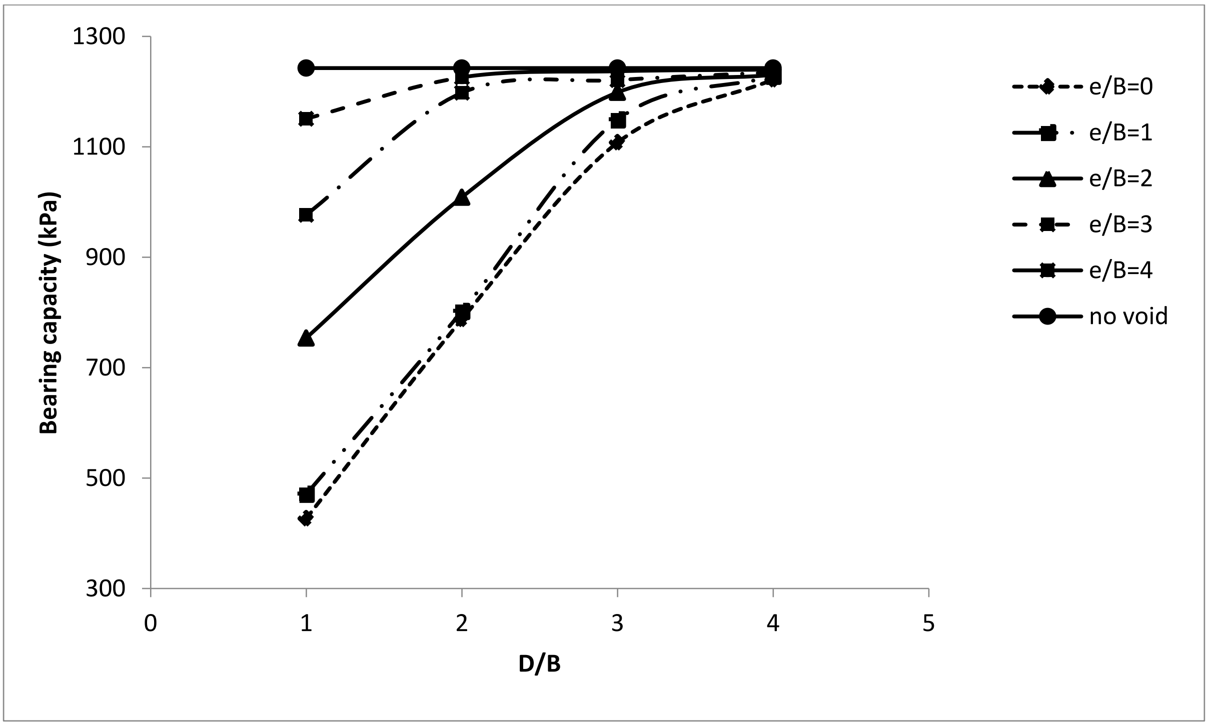



**Fig 4.13 variation of bearing capacity of footing with e/B and D/B**

### 4.2.1.3 Analysis of single void with variation in size of void

The behaviour of footing was analysed with the variation in size of void by considering w/B= 0.6, 1.2, 2.4, 3.6 where 'w' is diameter of circular void. Fig 4.14 shows the failure mechanism of soil with change in size of void (diameter of circular void). The failure mechanism resembles the failure pattern given by Wang et.al (1987). The failure mechanisms in figure 4.14, figure 4.15 and figure 4.16 resembles the failure mechanism A, B and C respectively. Fig 4.18 shows the bearing capacity of footing decreases with the increase in w/B ratio, for given constant values of zero eccentricity and D/B=2.The rate of decrease in bearing capacity increases first and the decreases with increase in w/B ratio.

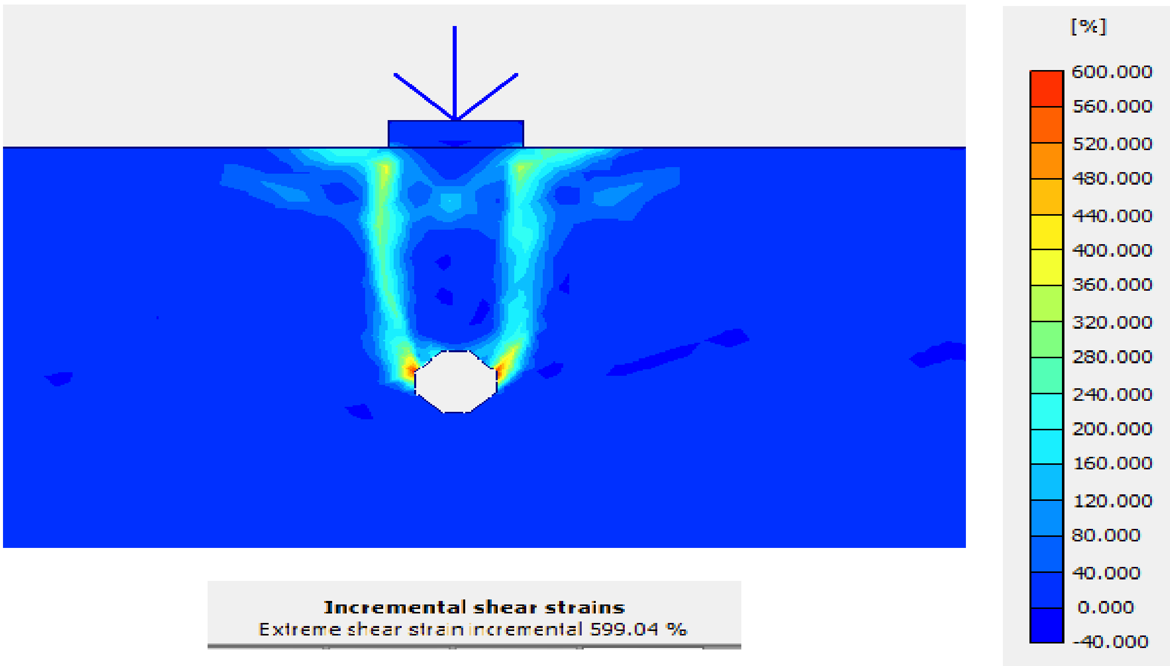



**Fig 4.14 Failure mechanism of soil with void (e/B=0; D/B=2; w/B=0.6)**

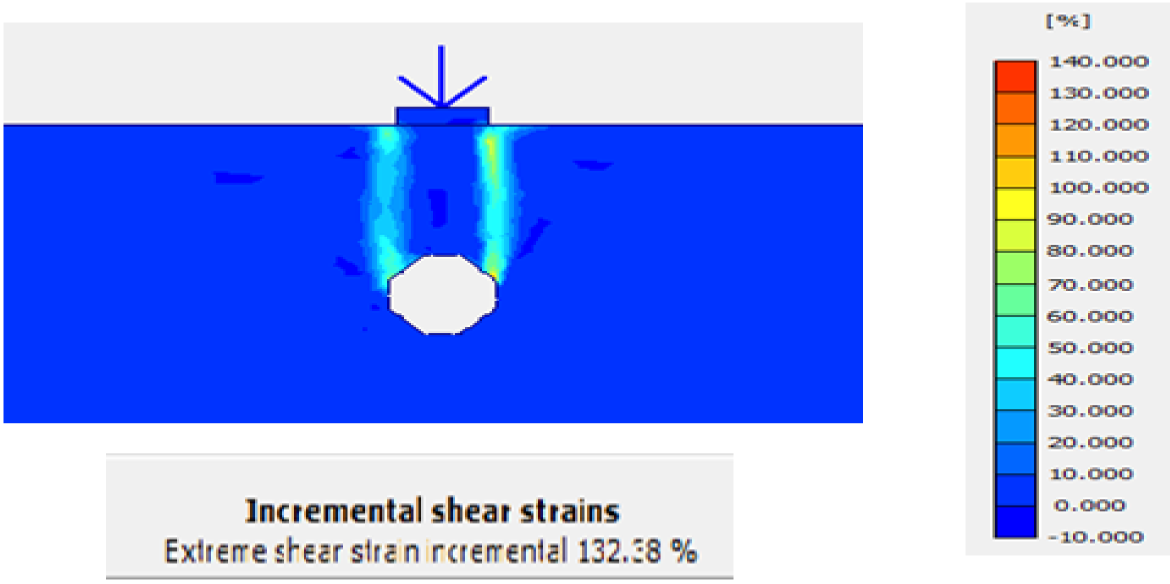



**Fig 4.15 Failure mechanism of soil with void (e/B=0; D/B=2; w/B=1.2)**

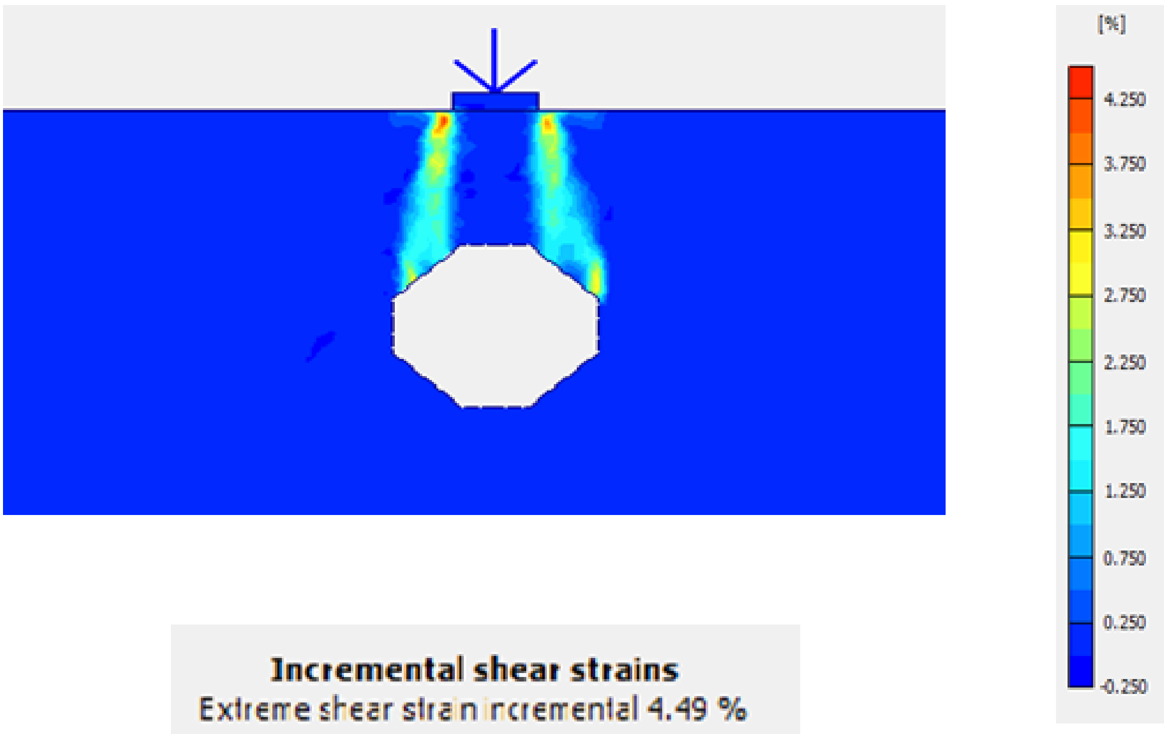



**Fig 4.16 Failure mechanism of soil with void ( e/B=0; D/B=2; w/B=2.4)**

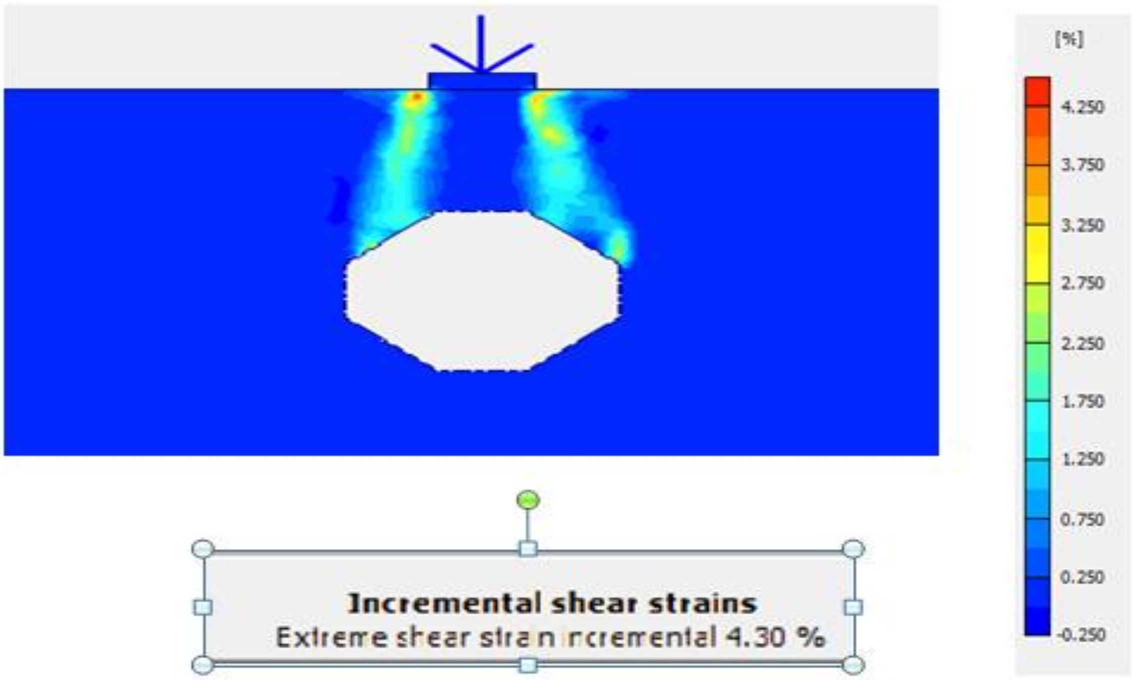



**Fig 4.17 Failure mechanism of soil with void ( e/B=0; D/B=2; w/B=3.6)**

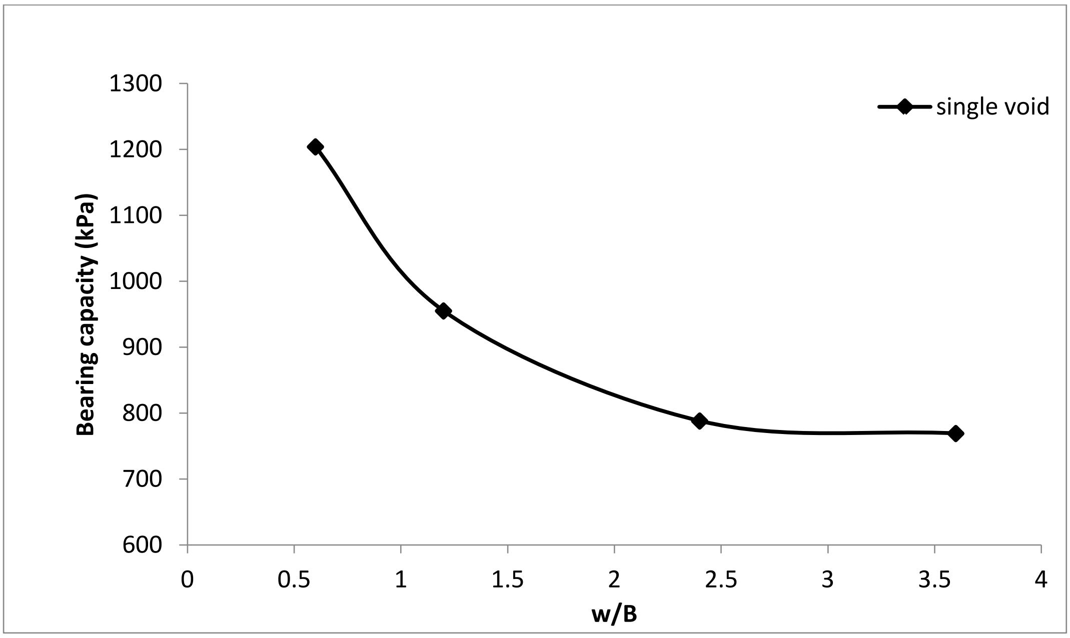



**Fig 4.18 Variation of bearing capacity with size of void**

#### 4.2.1.4 Analysis with Multiple voids

The variation of bearing capacity was analysed by considering multiple voids (double void). In present study two circular voids are considered with D/B=2; w/B=2.4 and e/B=1.5, 2, 3 and 4 on both sides of centre of footing. Fig 4.19 shows the double void considered in horizontal direction with eccentricity e=102 mm in other words e/B=2 (the distance between centre of footing to distance between centre of void on either side of footing).

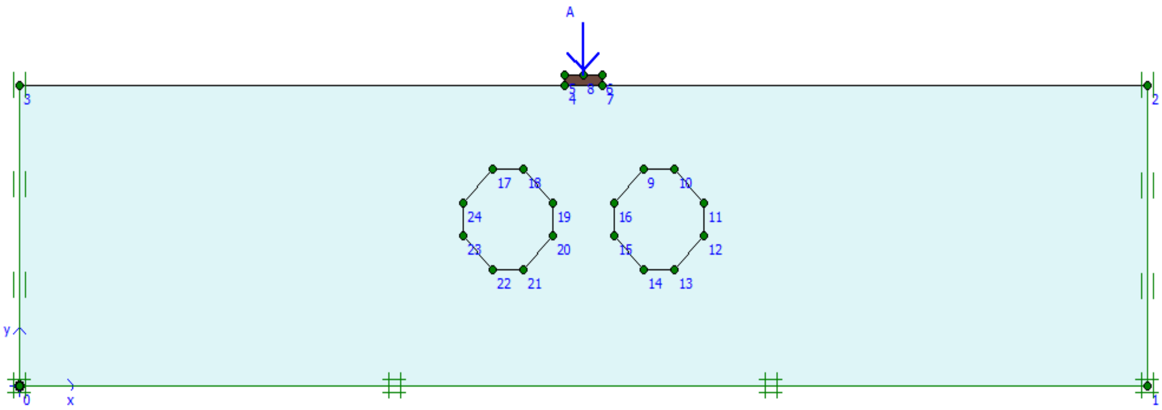



**Fig 4.19 Plaxis model of footing with two voids**

Fig 4.20 shows the failure mechanism of footing with two voids. Fig 4.21 shows the total displacements of soil after failure. Fig 4.21 shows the variation of bearing capacity of footing with increase in number of voids. With double void the bearing capacity is decreased compared to single void. The rate of decrease in bearing capacity decreases with increase in eccentricity. Here w/B=2.4, D/B=2 and zero depth of footing.

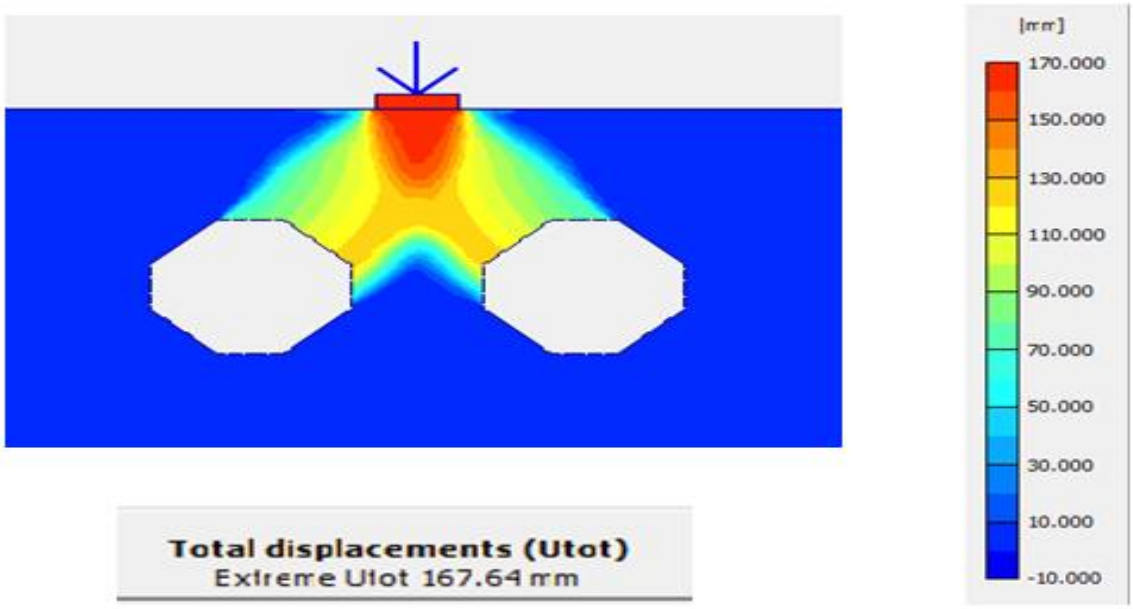



**Fig 4.20 Total displacements of soil with two voids**

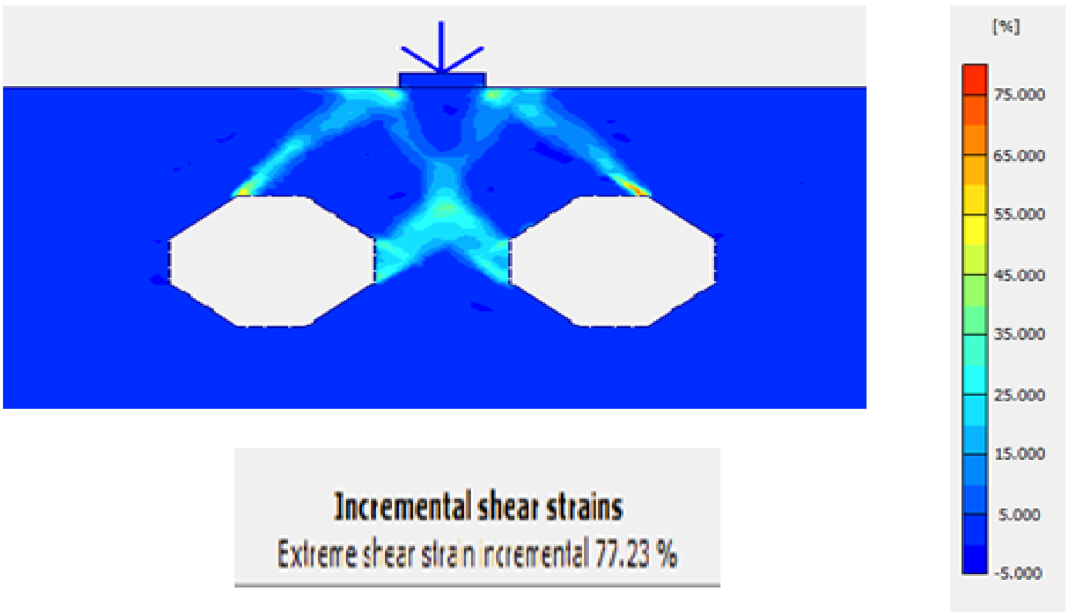



**Fig 4.21 Failure mechanism of footing with two voids**

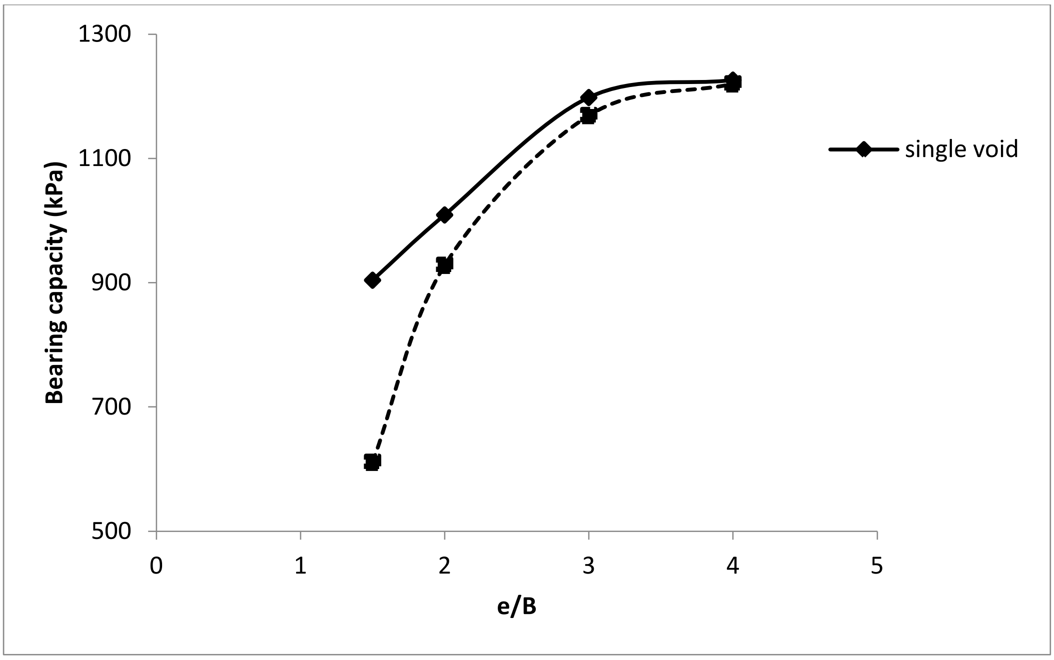



**Fig 4.22 Variation of bearing capacity with multiple voids**

### 4.2.1.5 Variation with Depth of Footing

The bearing capacity was calculated for no void and with void conditions by changing the depth of foundation. The ratio of depth of foundation and width of footing is expressed in the dimensional less constant value. In present study Df/B = 0, 1, 2, 3, 4 and 5 was considered for e/B=0; D/B=2 and w/B=2.4 for with void condition. Fig 5 shows the plaxis model of footing with depth of foundation 102mm. Fig 4.23 and 4.24 shows the total displacements of soil, the slip planes first developed below the footing and extends to ground surface. In fig 4.23 the arrows shows that the failure planes didn't extend to ground surface because confinement will increase with increase in depth of foundation. The direction of arrows shows the movement of soil particles. As the footing is located at certain depth delow the ground level punching shear failure takes place in which the soil body beneath the footing collapses into the void. Fig 4.24 shows the pressure bulb of footing over void for certain depth of footing.

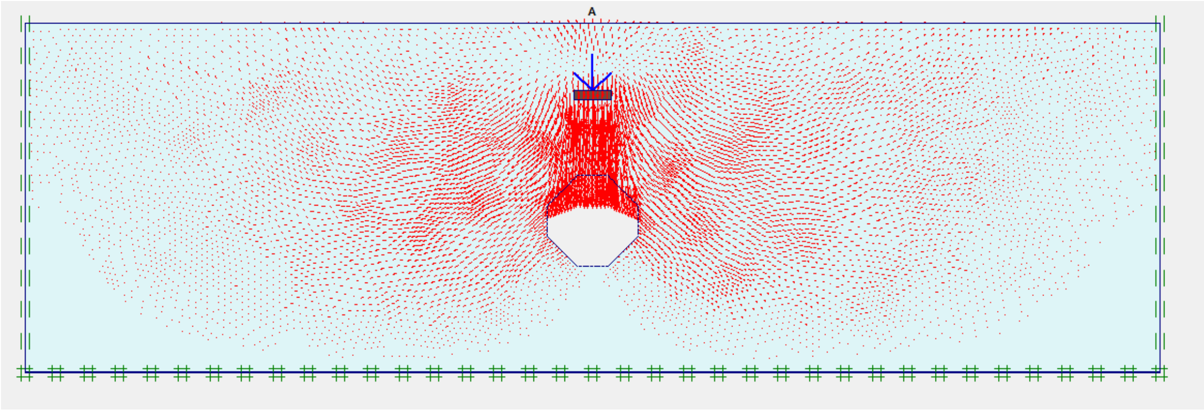



**Fig 4.23 Total displacements shown as arrows with increase in depth**

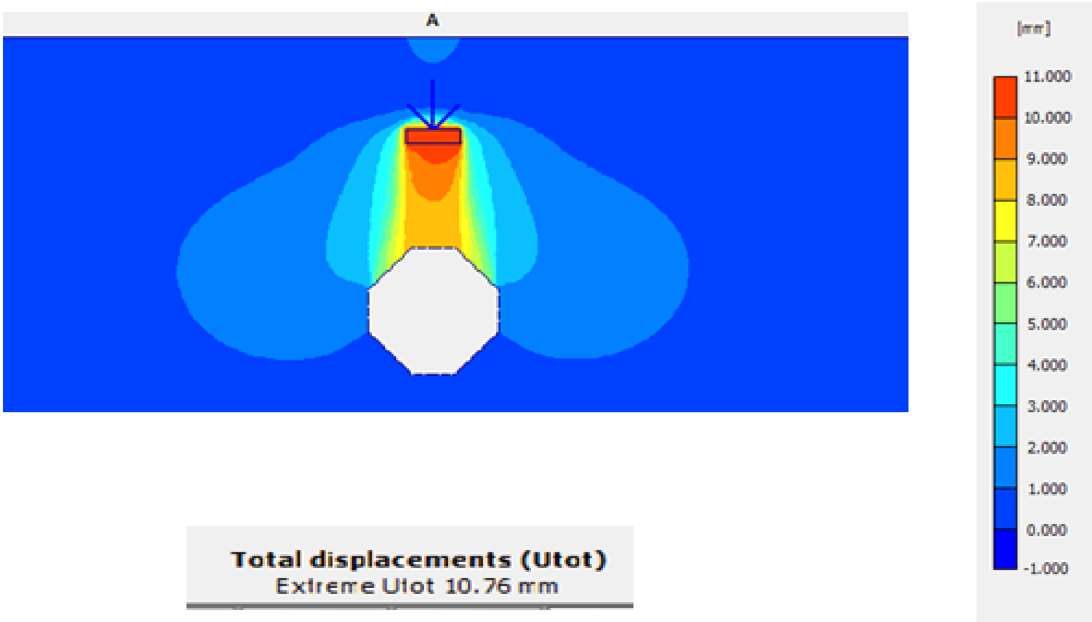



**Fig 4.24 Total displacements shown as shadings with increase in depth**

The variation of bearing capacity with depth of foundation is shown in fig 4.25 for no void and with void conditions. In both cases the bearing capacity increases with increase in depth of foundation as confinement increases so that the movement of soil particles towards ground surface reduces. There is a rapid increase in bearing capacity value when footing is placed at certain depth than the surface level. With further increase in depth of foundation increase in bearing capacity will not be affected by depth of footing. This study indirectly gives the depth of footing to be taken incase of void exists under footing.

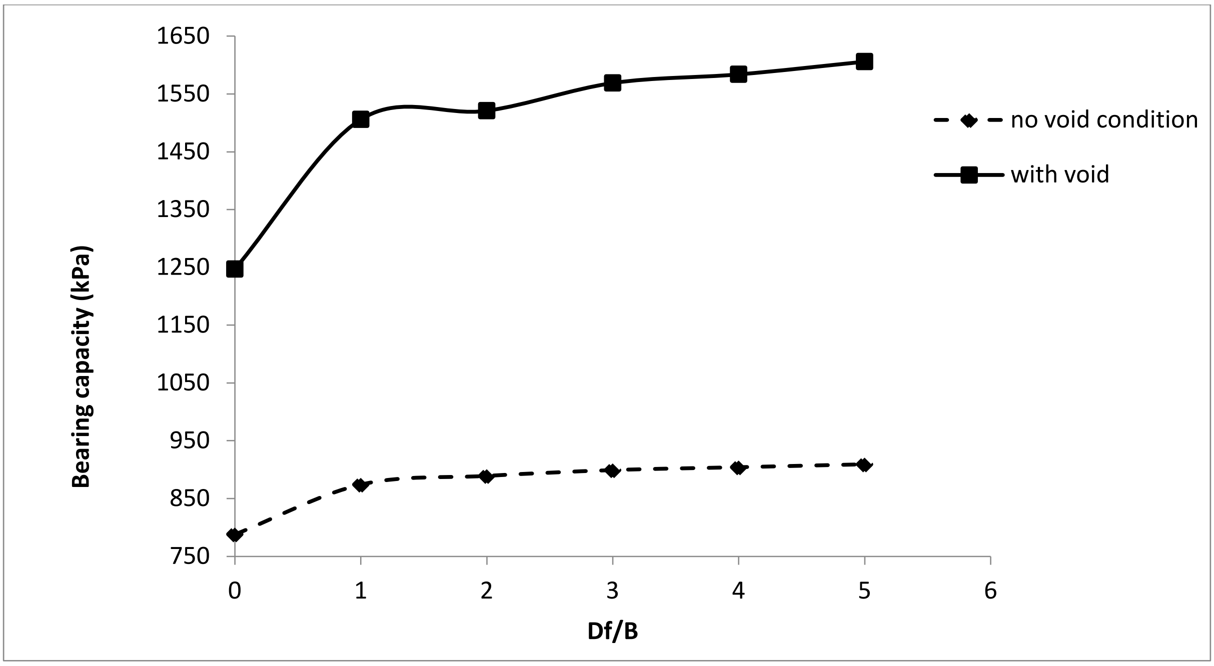



**Fig 4.25 Variation of bearing capacity with depth of footing**

### 4.2.2 SETTLEMENT CRITERIA

### 4.2.2.1 Variation of e/B and D/B

For a given constant load the settlement of footing was obtained with void at different locations. In present study void at different locations with e/B=0, 1, 2, 3 and 4 and D/B=1, 2, 3 and 4 was analysed for settlement of footing. Fig 4.26 shows the settlement decreases with increase in D/B ratio for a given load and e/B. The decrease in settlement decreases and finally it will reach settlement of no void condition with further increase in D/B. here w/B=2.4 and depth of foundation is zero.

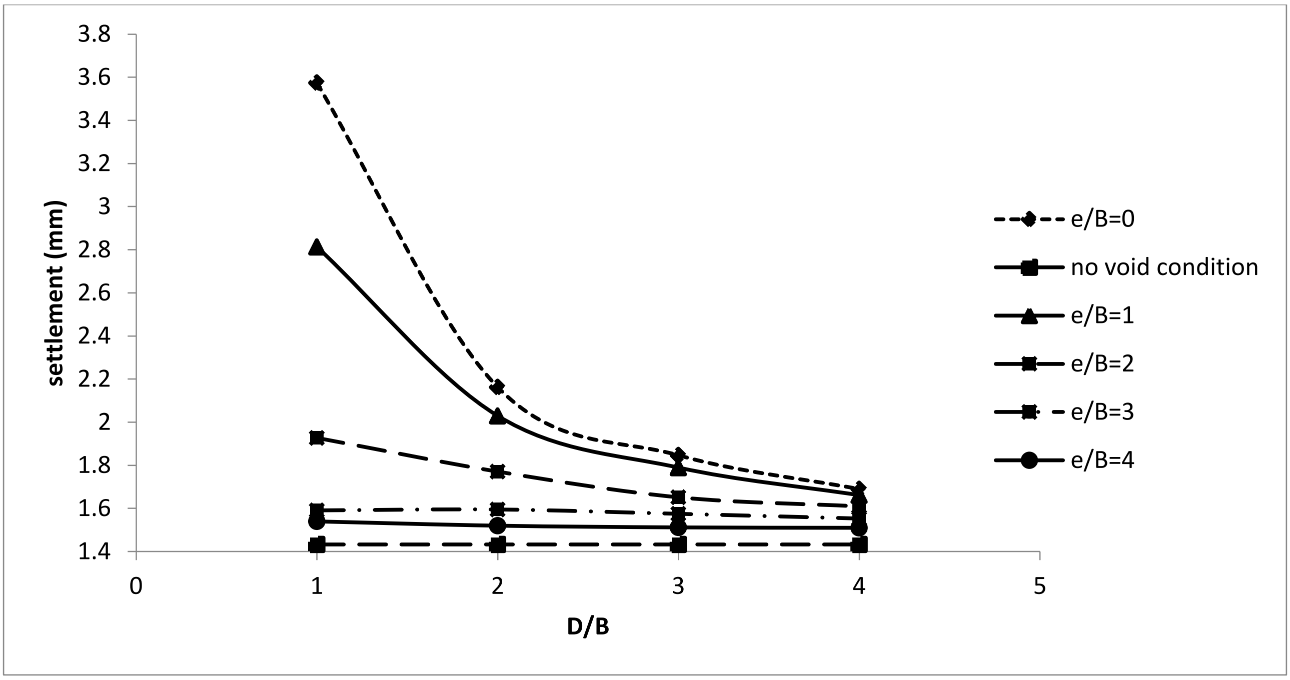



**Fig 4.26 Variation of settlement with e/B and D/B**

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### 4.2.2.2 Variation of Size of Void

The settlement of footing with change in diameter of circular void was analysed for a given value of D/B=2 and e/B=0 with w/B= 0.6, 1.2, 2.4 and 3.6. Fig 4.27 shows the settlement increases with increase in diameter of circular void at same location. With further increase in size of void the settlement will become constant.

### 4.2.2.3 Variation with Number of Voids

Fig 4.28 shows the settlement decreases with the increase in distance of void from centre of footing on both sides in horizontal direction for a constant void size and D/B. The settlement is more incase of footing with double void than single void.

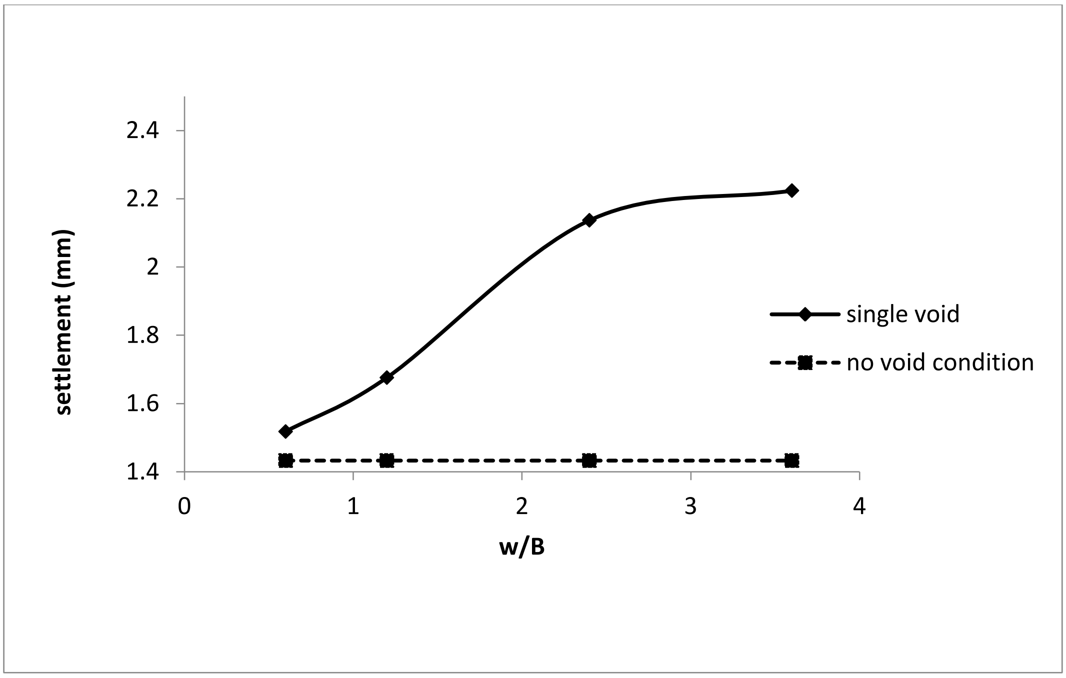


**Fig 4.27 Variation of settlement with size of void**

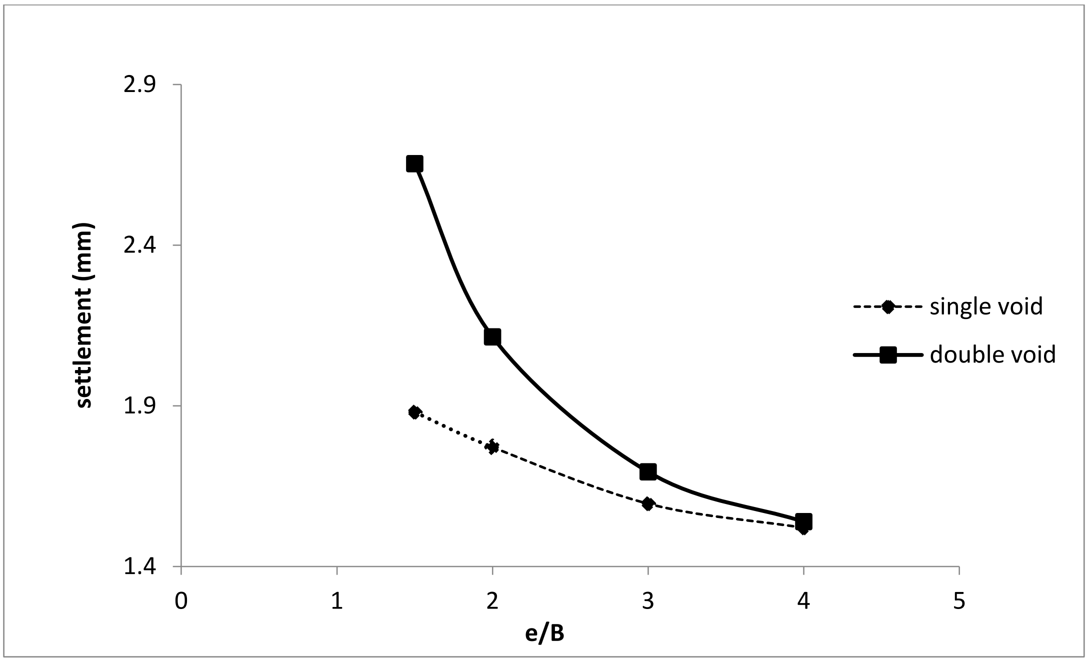


**Fig 4.28 Variation of settlement with number of voids**

### 4.2.2.4 Variation with Depth of Foundation

The settlement analysis was done by varying depth of footing. In this study $D_f$/B= 0, 1, 2, 3, 4 and 5 is considered. A constant load of 392 KN/mm was applied to get settlement of footing. Fig 4.29 shows the total displacements of soil for $D_f$/B=2, the soil particles didn't extend to ground surface due to increase in confinement.

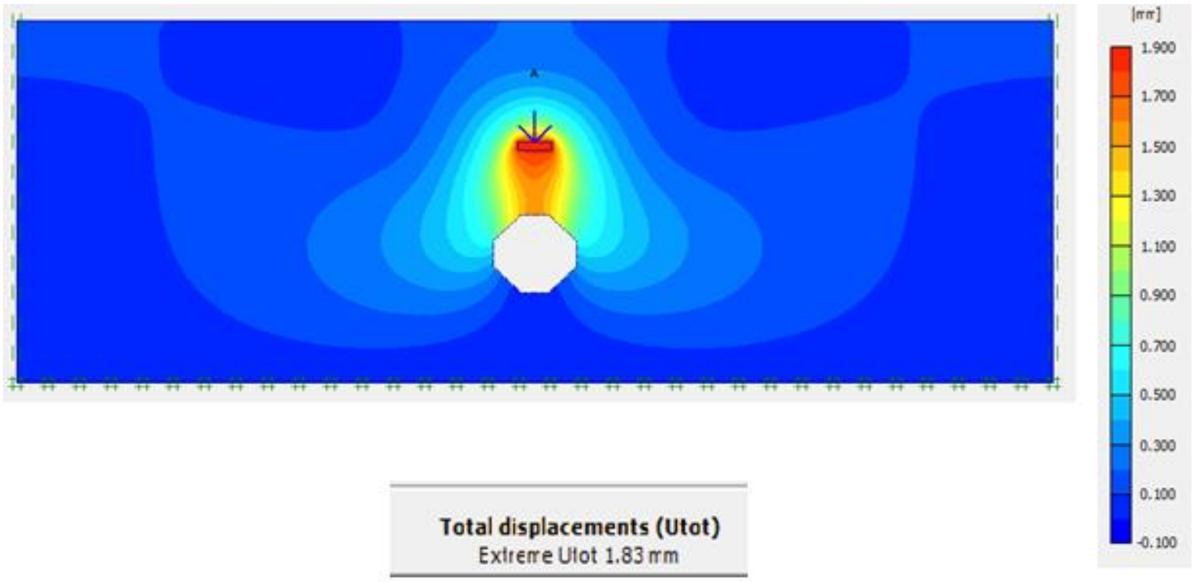


**Fig 4.29 Total displacements for Df/B=2 (settlement criteria)**

Fig 4.30 shows the settlement of footing without void and with void cases. The decrease in settlement is more when footing is placed at 51mm below the ground surface than the settlement with footing at the surface. With further increase in depth the rate of decrease in settlement decreases.

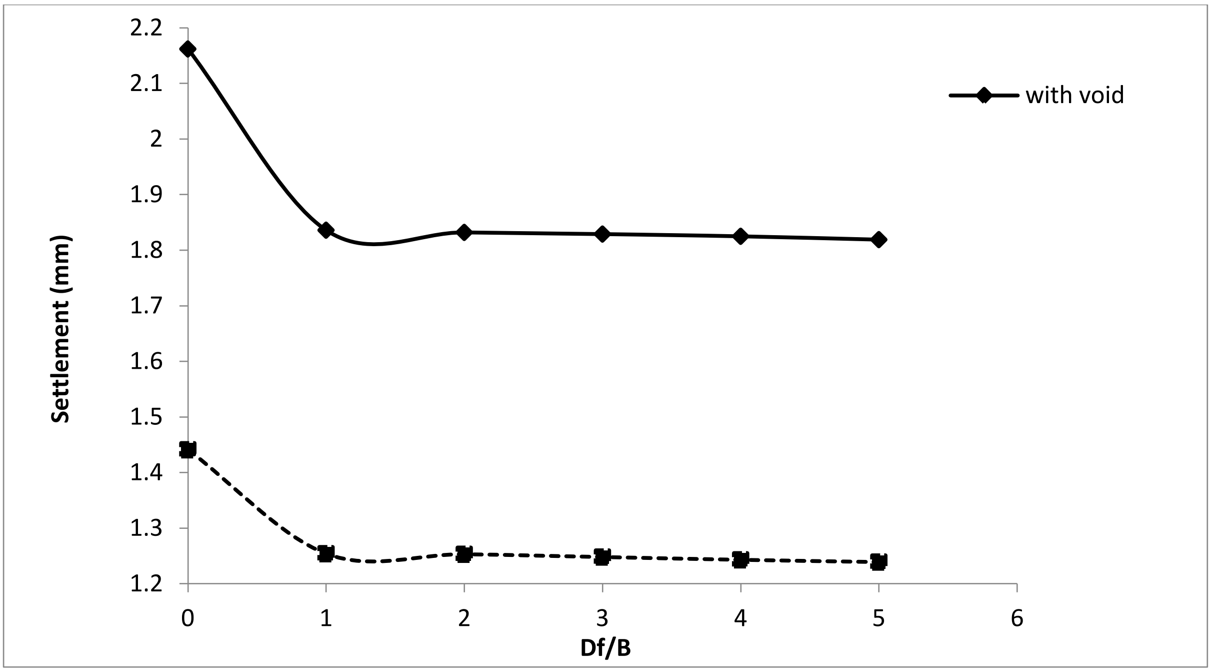


**Fig 4.30 settlement of footing with varying Df/B**

# CHAPTER 5

## 5. RELIABILITY ANALYSIS OF FOOTING WITH VOID

For reliability analysis five parameters of soil are considered as random variables.

**Table 3 Parameters with Mean and Standard deviation**

| parameter | Mean($\mu$) | Cov (%) | Standard deviation($\sigma$) |
|---|---|---|---|
| C | 158.7 | 20 | 31.74 |
| E | 19872 | 10 | 1987.2 |
| $\gamma$ | 13.23 | 7 | 0.9261 |
| $\nu$ | 0.39 | 13 | 0.0507 |
| $\phi$ | 8 | 5 | 0.4 |

Regression analysis has performed using least square error method. The developed response surface model is used for determining the reliability index ($\beta$).

The considered parameters are uncorrelated normally distributed parameters with the lower limit ($\mu$+2$\sigma$) and upper limit ($\mu$–2$\sigma$) to quantify each point in design sets from normal distribution.

Full factorial design:

Matlab code used for design is

```
>> dFF2=ff2n(5)

dFF2 =

     0     0     0     0     0
     0     0     0     0     1
     0     0     0     1     0
     0     0     0     1     1
     0     0     1     0     0
     0     0     1     0     1
     0     0     1     1     0
     0     0     1     1     1
     0     1     0     0     0
     0     1     0     0     1
     0     1     0     1     0
     0     1     0     1     1
     0     1     1     0     0
     0     1     1     0     1
     0     1     1     1     0
     0     1     1     1     1
     1     0     0     0     0
     1     0     0     0     1
     1     0     0     1     0
     1     0     0     1     1
     1     0     1     0     0
     1     0     1     0     1
     1     0     1     1     0
```

| | | | | |
|---|---|---|---|---|
| 1 | 0 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 0 | 0 | 1 |
| 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 |

## 5.1 SINGLE VOID

**Table 4 Settlement of footing over Single void corresponding to 32 sample points in RSM using Plaxis**

| | C ($KN/m^2$) | $\gamma$ ($KN/m^3$) | $\phi$ | E ($KN/m^2$) | $\nu$ | $\delta$ |
|---|---|---|---|---|---|---|
| $\mu+2\sigma$ (0) | 222.18 | 15.08 | 8.8 | 23846.4 | 0.49 | |
| $\mu-2\sigma$ (1) | 95.22 | 11.38 | 7.2 | 15897.6 | 0.29 | |
| 1 | 222.18 | 15.08 | 8.8 | 23846.4 | 0.49 | 1.84 |
| 2 | 222.18 | 15.08 | 8.8 | 23846.4 | 0.29 | 2.58 |
| 3 | 222.18 | 15.08 | 8.8 | 15897.6 | 0.49 | 2.76 |
| 4 | 222.18 | 15.08 | 8.8 | 15897.6 | 0.29 | 3.88 |
| 5 | 222.18 | 15.08 | 7.2 | 23846.4 | 0.49 | 1.84 |
| 6 | 222.18 | 15.08 | 7.2 | 23846.4 | 0.29 | 2.59 |
| 7 | 222.18 | 15.08 | 7.2 | 15897.6 | 0.49 | 2.76 |
| 8 | 222.18 | 15.08 | 7.2 | 15897.6 | 0.29 | 3.88 |
| 9 | 222.18 | 11.38 | 8.8 | 23846.4 | 0.49 | 1.84 |
| 10 | 222.18 | 11.38 | 8.8 | 23846.4 | 0.29 | 2.58 |
| 11 | 222.18 | 11.38 | 8.8 | 15897.6 | 0.49 | 2.76 |
| 12 | 222.18 | 11.38 | 8.8 | 15897.6 | 0.29 | 3.88 |
| 13 | 222.18 | 11.38 | 7.2 | 23846.4 | 0.49 | 1.84 |
| 14 | 222.18 | 11.38 | 7.2 | 23846.4 | 0.29 | 2.68 |
| 15 | 222.18 | 11.38 | 7.2 | 15897.6 | 0.49 | 2.86 |
| 16 | 222.18 | 11.38 | 7.2 | 15897.6 | 0.29 | 3.98 |
| 17 | 95.22 | 15.08 | 8.8 | 23846.4 | 0.49 | 4.51 |
| 18 | 95.22 | 15.08 | 8.8 | 23846.4 | 0.29 | 5.96 |
| 19 | 95.22 | 15.08 | 8.8 | 15897.6 | 0.49 | 6.77 |
| 20 | 95.22 | 15.08 | 8.8 | 15897.6 | 0.29 | 8.94 |
| 21 | 95.22 | 15.08 | 7.2 | 23846.4 | 0.49 | 6.68 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 22 | 95.22 | 15.08 | 7.2 | 23846.4 | 0.29 | 7.81 |
| 23 | 95.22 | 15.08 | 7.2 | 15897.6 | 0.49 | 10.03 |
| 24 | 95.22 | 15.08 | 7.2 | 15897.6 | 0.29 | 11.73 |
| 25 | 95.22 | 11.38 | 8.8 | 23846.4 | 0.49 | 4.47 |
| 26 | 95.22 | 11.38 | 8.8 | 23846.4 | 0.29 | 5.95 |
| 27 | 95.22 | 11.38 | 8.8 | 15897.6 | 0.49 | 6.71 |
| 28 | 95.22 | 11.38 | 8.8 | 15897.6 | 0.29 | 8.93 |
| 29 | 95.22 | 11.38 | 7.2 | 23846.4 | 0.49 | 6.64 |
| 30 | 95.22 | 11.38 | 7.2 | 23846.4 | 0.29 | 7.74 |
| 31 | 95.22 | 11.38 | 7.2 | 15897.6 | 0.49 | 9.96 |
| 32 | 95.22 | 11.38 | 7.2 | 15897.6 | 0.29 | 11.61 |

From the regression analysis the equation for settlement is

$\delta$ = 25.61 - 0.03932833*C--0.000266471*E +0.002195946*$\gamma$–0.791796875*$\phi$–6.390625* $\nu$

The performance of the function is given by F(x) = X - $\delta$

Here X = 40 mm (allowable settlement)

Minimize $\beta = F(X*) = 0^{min}\sqrt{(X*)^t(X*)}$

$$X* = \frac{x-\mu}{\sigma}$$

Initially the value of x is equal to mean value of random variable. Using Excel solver the value of β is obtained from iterative procedure. After the iterative process the minimum value of β is obtained.

The minimum distance from the origin to design point is β=4.96.

From MS-Excel $P_f$ = NORMSDIST (-β)

The probability of failure of footing is $p_f = 3.52 \times 10^{-7}$ and from the USACE chart the footing is good for given loading conditions.

## 5.2 DOUBLE VOID

**Table 5 Settlement of footing over Double void corresponding to 32 sample points in RSM using Plaxis**

| | C ($KN/m^2$) | $\gamma$ ($KN/m^3$) | $\phi$ | E ($KN/m^2$) | $\nu$ | Settlement $\delta$ (mm) |
|---|---|---|---|---|---|---|
| $\mu+2\sigma$ (0) | 222.18 | 15.08 | 8.8 | 23846.4 | 0.49 | |
| $\mu-2\sigma$ (1) | 95.22 | 11.38 | 7.2 | 15897.6 | 0.29 | |
| 1 | 222.18 | 15.08 | 8.8 | 23846.4 | 0.49 | 2.18 |
| 2 | 222.18 | 15.08 | 8.8 | 23846.4 | 0.29 | 3 |
| 3 | 222.18 | 15.08 | 8.8 | 15897.6 | 0.49 | 3.26 |
| 4 | 222.18 | 15.08 | 8.8 | 15897.6 | 0.29 | 4.51 |
| 5 | 222.18 | 15.08 | 7.2 | 23846.4 | 0.49 | 2.18 |
| 6 | 222.18 | 15.08 | 7.2 | 23846.4 | 0.29 | 3.01 |
| 7 | 222.18 | 15.08 | 7.2 | 15897.6 | 0.49 | 3.26 |
| 8 | 222.18 | 15.08 | 7.2 | 15897.6 | 0.29 | 4.52 |
| 9 | 222.18 | 11.38 | 8.8 | 23846.4 | 0.49 | 2.17 |
| 10 | 222.18 | 11.38 | 8.8 | 23846.4 | 0.29 | 3 |
| 11 | 222.18 | 11.38 | 8.8 | 15897.6 | 0.49 | 3.26 |
| 12 | 222.18 | 11.38 | 8.8 | 15897.6 | 0.29 | 4.5 |
| 13 | 222.18 | 11.38 | 7.2 | 23846.4 | 0.49 | 2.17 |
| 14 | 222.18 | 11.38 | 7.2 | 23846.4 | 0.29 | 3.01 |
| 15 | 222.18 | 11.38 | 7.2 | 15897.6 | 0.49 | 3.26 |
| 16 | 222.18 | 11.38 | 7.2 | 15897.6 | 0.29 | 4.51 |
| 17 | 95.22 | 15.08 | 8.8 | 23846.4 | 0.49 | 158.69 |
| 18 | 95.22 | 15.08 | 8.8 | 23846.4 | 0.29 | 133.58 |
| 19 | 95.22 | 15.08 | 8.8 | 15897.6 | 0.49 | 231.23 |
| 20 | 95.22 | 15.08 | 8.8 | 15897.6 | 0.29 | 200.61 |
| 21 | 95.22 | 15.08 | 7.2 | 23846.4 | 0.49 | 354.17 |
| 22 | 95.22 | 15.08 | 7.2 | 23846.4 | 0.29 | 77.12 |
| 23 | 95.22 | 15.08 | 7.2 | 15897.6 | 0.49 | 537.69 |
| 24 | 95.22 | 15.08 | 7.2 | 15897.6 | 0.29 | 116.2 |
| 25 | 95.22 | 11.38 | 8.8 | 23846.4 | 0.49 | 148.05 |
| 26 | 95.22 | 11.38 | 8.8 | 23846.4 | 0.29 | 52.6 |
| 27 | 95.22 | 11.38 | 8.8 | 15897.6 | 0.49 | 226.24 |
| 28 | 95.22 | 11.38 | 8.8 | 15897.6 | 0.29 | 78.93 |
| 29 | 95.22 | 11.38 | 7.2 | 23846.4 | 0.49 | 376.91 |
| 30 | 95.22 | 11.38 | 7.2 | 23846.4 | 0.29 | 79.03 |
| 31 | 95.22 | 11.38 | 7.2 | 15897.6 | 0.49 | 565.36 |
| 32 | 95.22 | 11.38 | 7.2 | 15897.6 | 0.29 | 139.38 |

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From the regression analysis the equation for settlement is

$\delta$ = 568 -1.68556534*C-0.00570723*E -2.41266892*$\gamma$ -39.6863281*$\phi$ + 535.178125* $\nu$

The performance of the function is given by G(x) = X*- $\delta$

Minimize $\beta = G(X*) = 0^{min}\sqrt{(X*)^t(X*)}$

$$X* = \frac{x-\mu}{\sigma}$$

Initially the value of x is equal to mean value of random variable. Using Excel solver the value of $\beta$ is obtained from iterative procedure. After the iterative process the minimum value of $\beta$ is obtained.

The minimum distance from the origin to design point is $\beta$=0.995.

From MS-Excel $P_f$ = NORMSDIST (-$\beta$)

The probability of failure of footing is $p_f$ = 0.16 and from the USACE chart the footing is hazardous for given loading conditions.

# CHAPTER 6

## 6.1 CONCLUSION

In present study the stability of spread footing over continuous circular void under different boundary conditions such as void size, location of void, number of voids has been done using finite element method (PLAXIS 8.1) and results are compared with available literature. The reliability analysis of structure has been done using first order reliability method (FORM). The probability of failure of structure under standard conditions is found out using USACE (1997) chart. Response surface method is used for generating the limit state function. Conclusions from the present study are described as follows:

1. The bearing capacity of footing increases with increase in void depth from base of footing and eccentricity. The failure of footing occurred when the void is located immediately below the footing, known as critical region. With the increase in eccentricity and void depth, the region of void is outside of pressure bulb therefore bearing capacity increases. The bearing capacity from model testing with no void condition is 1250 kPa and from FEM its value is 1243 kPa. The error is due to uncontrolled parameters during experiment or during simulation in FEM. The percentage error is 0.56%, therefore FEM gives reasonable results and can be used in practical applications. Settlement of footing decreases with increase in eccentricity and depth of void, as the void is far away from the pressure bulb.
2. Bearing capacity decreases with increase in void size as the soil collapses into void results in punching shear failure. settlement increases with increase in void size
3. Analysis of multiple voids has been done. The decrease in bearing capacity is almost half of that single void. The settlement with double increases more than 30% than that off single void.

4. With the increase in depth of foundation the bearing capacity increases and settlement decreases as the confinement increases. The failure planes starts from below the base of footing and extension of slip lines towards surface decreases. The failure mechanisms from FEM are compared with mechanism given by Wang et.al
5. Presence of uncertainties results failure of structure which are not found using deterministic approach. Therefore reliability analysis was performed, using first order reliability method, the reliability index for footing over single void is found as 4.96 and probability of failure of structure is found as $3.52 \times 10^{-7}$ From USACE (1997) chart the structure is good under standard conditions. The reliability index for footing over double void is found as 0.995 and probability of failure of structure is found as 0.16. From USACE (1997) chart the structure is Hazardous under standard conditions.

## 6.2 SCOPE FOR FUTURE STUDY

Based on the present study it was observed that void below footings cause severe affects on its stability. Therefore the preventive measures to reduce the effect of voids on structure should be studied. Usage of geomaterials like geogrids reduce the effect considerably, it should be analysed using Experimental, analytical and numerical methods. Even though deterministic analysis gives better results the probability of structure to fail is unknown due to variability in properties and other conditions. Therefore usage of reliability gives idea about this. Besides using normal distribution reliability should be performed using other distribution.

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